Brief Notes on Logic & Related Stuff

1. What is logic?

Logic, as a branch of philosophy, is the theory of *correct reasoning*. It studies the general nature of reasoning and the criteria for determining whether a piece of reasoning is valid or not.

Logic is very important to philosophy because philosophers are concerned with providing rational arguments for their views. Providing rational arguments, in turn, is important because the answers to most of the questions we consider are not immediately obvious, so they require some thought process to discover. If someone thinks he knows the answer to a philosophical question, then, he will have to explain the reasoning process that leads him to that answer; and this will generally amount to an argument.

2. What is a proposition?

There are three kinds of things that can be either true or false: statements, beliefs, and propositions.

(a) A *statement* is a certain kind of sentence, which is a sequence of physical symbols. For instance, the sentence, "All dogs are furry" is a statement; it can be either true or false. But sentences such as "What time is it?" and "Please turn the music down" are not statements, since they cannot be either true or false.

(b) A *belief* (or judgement, or opinion) is a kind of mental state that can be either true or false, and that would be expressed (if you express it at all) by a statement. For instance, if you think that all dogs are furry, then your belief is either true or false. If you want to express your belief, you will utter the statement, "All dogs are furry."

On the other hand, such mental states as emotions, sensations, and desires are neither true nor false. For instance, if you are upset that your dog pooped on the carpet, your feeling of anger is not itself either 'true' or 'false'. You might express your feeling by saying something like "Damnit!" (not a statement).

(c) The term "proposition" has more than one meaning in English, but I propose to use it like this: A proposition is the sort of thing that a belief or statement is *about*. That is: when you have a belief, there is a way that you think the world is, a state of affairs that you think obtains. If that state of affairs does obtain, then your belief is true; if not, then your belief is false. A *proposition*, then, is a possible state of affairs, i.e. *a way the world might be*. A proposition is either true or false (it is true if the world is that way; false if the world is not that way).

Notice that propositions, unlike statements and beliefs, do not depend on us for their existence. Even before there were living things, it was true, for example, that the Earth orbited the Sun (i.e., there was the *fact* that the Earth orbited the Sun -- the fact, unlike beliefs or statements about it, is independent of us).

To see that propositions are different from statements, compare the two statements, "It is raining" and "Il pleut" (which is the French translation of "It is raining"). These are certainly *different statements* (different sequences of words). But equally clearly, they have something important in common. What? They *express the same proposition* -- that is, they both describe the same possible state of affairs.

3. What is an argument?

An argument is a series of statements that tries to present good reasons for believing something. That is, it contains a series of statements, one of which is supposed to follow from the others. Components of arguments:

(a) Every argument has a *conclusion:* this is the claim you are arguing for, the thing you are trying to convince someone of.

(b) Every argument (except for some "*reductio ad absurdum"* arguments, which we won't talk about) has one or more *premises:* These are the reasons that you are presenting to support the conclusion.

(c) An argument may have one or more *intermediate steps*. These are steps in between the premises and the conclusion that help you to see how the conclusion follows from the premises. The intermediate steps are supported by the premises, and in turn support the conclusion.

4. Validity & soundness

In logic, "valid" is used as a technical term. To say an argument is *valid* is to say that its conclusion follows from its premises. In other words, to say an argument is valid means just this: *if* the premises were true, *then* the conclusion would have to be true. So it would not be possible for the premises to be true and the conclusion to be false.

It's possible for an argument to be 'valid' in this sense even if the premises are false, because "validity" only refers to the *relationship* between the premises and the conclusion.

On the other hand, an argument is said to be 'sound' if it is valid *and* the premises are also true.

Examples:

1. "Socrates is a fish; all fish are mammals; therefore, Socrates is a mammal" is a *valid* argument: that is, the conclusion follows from the premises. However, it is not *sound*, because both premises are false.

2. "Socrates is a man; all men are mortal; therefore, Socrates is mortal" is both valid and sound.

3. "Socrates is a man; all men are mortal; therefore, the moon is made of green cheese" is neither valid nor sound.

Note:

Never call an argument 'false'. Only an individual *proposition* can be either 'true' or 'false'. *Never call a proposition 'valid' or 'sound'.* Only a whole *argument* can be valid, invalid, sound, or unsound.

5. Symbolism

The following symbols are typically used in philosophy to stand for propositions. Consequently, it's important to know what they mean:

A — Used to stand for any proposition (other letters can also be used, of course.) E.g., "All cats are fluffy."

¬A — This means, "It is not the case that A." E.g., "Not all cats are fluffy."

(A ∨ B) — This is read "Either A or B (or both)". E.g., "Either the light bulb is burned out, or the lamp doesn't work (or both)."

(A & B) — Read, "A and B". E.g., "All cats are fluffy, and some cats are ugly."

(A → B) — Read, "If A then B". E.g., "If all cats are fluffy, then some cats are ugly."

(A ↔ B) — Read "A if and only if B". E.g., "All cats are fluffy if and only if some cats are ugly."

Example:
Consider the argument, "Either Bob or Ted stole money from the company. Bob didn't do it. Therefore, Ted did it." This can be symbolized like so:

(1) (B ∨ T).
(2) ¬B.
(3) ∴ T.

(where B=Bob stole money from the company, and T=Ted stole money from the company). This is a valid argument.

6. Characteristics of propositions

We've already said that propositions have the characteristic of being either true or false. Propositions, statements, and beliefs are the *only* things that can be either true or false. They are also the only things that can be *proved*, *refuted*, *confirmed*, or *contradicted*, and they are the only things that can figure in arguments. Thus, for example, it makes no sense to speak of 'proving rationality': rationality is not a proposition; it's a character trait. Similarly, you can't argue against your emotions; emotions are not propositions.

Propositions can also have several other characteristics which are interesting and philosophically important:

(a) Modality:

Every proposition is either possible or impossible, and necessary or contingent:

- "possible": This term is usually used in a very broad sense (what we call "logical possibility"). A proposition is logically possible if there is at least some conceivable state of affairs that would have made it true. Example: "There are fewer than 9 planets in the solar system" is *logically possible* (even though we know it's false).

- "impossible": This term is used in the same broad sense. A proposition is said to be 'logically impossible' if it is contradictory, or for some other reason there is no conceivable way it could have been true. Examples: "There are fewer than 9 and more than 8 planets in the solar system" is logically impossible, since no matter how many planets there had been that proposition still would have been false. "The number seven is on fire" is also logically impossible, because it just doesn't make any sense.

- "necessary": A proposition is 'necessary' if no state of affairs would have made it false. In other words, to say that A is necessary means that ¬A is impossible. Example: "Either there are fewer than 9 or there are more than 8 planets in the solar system" is necessary, since any number of planets would have been either less than 9 or more than 8.

- "contingent": This means "not necessary but not impossible either". Example: "There are 9 planets in the solar system" is a contingent fact.

The "modality" of a proposition refers to which of the above characteristics a proposition has (a proposition can have more than one, e.g., contingent and possible).

(b) Analytic vs. synthetic:

Every proposition is either analytic or synthetic. A proposition is said to be "analytic" if it is true by definition; equivalently, a proposition is analytic if to deny it would be contradictory. E.g., "All bachelors are unmarried" is analytic, because "bachelor" just means "unmarried man"; it would therefore be contradictory to say that some bachelors are married.

"Synthetic" just means "not analytic". E.g., "Most bachelors are slobs" is synthetic, because being a slob isn't part of the definition of a bachelor.

(c) Empirical vs. a priori:

Some propositions can be known empirically, and others can be known *a priori*. Empirical knowledge is knowledge which is based on experience, or observation, either directly or indirectly. My knowledge that the sky is blue is empirical. Our knowledge of atomic theory is also empirical.

A priori knowledge is knowledge which is not at all based on observation. Those who believe that *a priori* knowledge exists say that mathematics and most of philosophy are bodies of *a priori* knowledge.

Questions:

1. Translate each of the following arguments into symbolic notation, and then say whether it is valid or not. (see section 4)

(a) If God doesn't exist, then the universe is the result of blind chance. The universe is not the result of blind chance. Therefore, God exists.

(b) My cat can speak French. My cat can speak English. Therefore, my cat can speak French and English.

2. For each of the following propositions, state whether it is analytic or synthetic.

(a) Cats are pretty.

(b) I like ice cream.

(c) It's not over til it's over.

(d) Every square has four sides.

3. For each of the following propositions, determine whether it is impossible, necessary, or contingent:

(a) The Empire State Building is the tallest building in the world.

(b) 2 is the square root of seven.

(c) If a=b and b=c, then a=c.

(d) The earth orbits the sun.