Optimization and Control of Networks

Optimization, Dynamics, and Layering in Complex Networked Systems: From the Internet to the Smart Grid



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Networked systems

Complexity is ever increasing

- Large in size and scope
- Enormous heterogeneity
- Incomplete information
- Uncertain environments
- Emerging technologies
- New applications

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New design dimensions

Design (& understanding) is increasingly dominated by

- Efficiency (optimality)
- Manageability
- Reliability & Security
- Economic viability
- Scalability
- Evolvability

□

emerging, collective properties



 Most persistent/shared organizational structure, including abstractions, interfaces, and layering of cyber and physical

Highly conserved resource
 allocation, control, and
 management mechanisms

Components

architecture

Components

Constraints that deconstrain

- Certain fixed points and structure under which the network can expand/evolve
- Constraining for the issues that the network was
 originally not designed for

good architecture enables innovation, bad one freezes it

architecture

Architectural design

Remains an art, primarily empirical, reasoning-based

Good architecture easy to recognize in retrospect but elusive to forward-engineer

Components

No formal theory nor systematic design method

A holy grail

Mathematical underpinning of network architecture



 Systematic methods to develop and evaluate design choices and algorithms

Approach

Understand architecture and main mechanisms of existing networks (reverse engineering)

architecture

Components

Design architecture and main mechanisms for emerging networks (forward engineering)

Computer

Software **Diverse**

Constraints
thatLayered
architecture
(toy model)



Communication system



Internet



Internet architecture



Emerging networked systems

future internet

energy-efficient data center

smart grid







architectures are being designed now ...

Future Internet architecture

Clean slate Internet design that aims to build in or enable

- security
- mobility

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- new communication paradigms
- new computing paradigms



not clear what the right architecture is and how to best design different components and their interactions

Energy-efficient data center

- How to decompose & coordinate energy management decisions spatially and temporally
- How to interact with other resource allocation algorithms
- How to interconnect servers to balance performance and energy usage



3-5% of total US energy use

theories and models are needed to guide architecture and algorithm design

Smart grid

Power network will go through similar architectural transformation in the next few decades that telephone network has gone through



Smart grid

... to become more distributed, more open, more interactive, more autonomous, and with greater user participation



what is an architecture theory to help guide the transformation?

... while maintaining security & reliability

architecture

Components

Research

- Rigorous foundations and new methodologies for
 understanding & designing architecture and various mechanisms
 - Employ and develop techniques in
 - optimization theory/algorithm
 - distributed control
 - **game theory**
 - systems theory

Approach



- must be abstract and concrete
- must be foundational and practical

Research

Architecture theory (foundations and design methodologies)

 wireless network

 network coding

 EE data center

 smart grid

networks and protocols & dynamics as distributed decomposition of optimization

Agenda

- Layering and constrained optimization (communication network)
- Network dynamics as optimization algorithms (power network)
- Look into future

Layered Internet protocol stack

Each layer

- controls a subset of decision variables
- hides the complexity of the layer below
- provides a service to the layer above
- designed independently and evolves asynchronously



Optimization and layering

- Each layer is abstracted as an optimization problem
- Operation of a layer is a distributed solution
- Results of one problem (layer) are parameters of others
- Operate at different timescales



Optimization and layering

Networks as optimizers

- integrate various protocol layers, by regarding them as carrying out distributed computation over the network to implicitly solve a certain global optimization problem
- different layers iterate on different subsets of decision variables using local information to achieve individual optimality
- taken together, these local algorithms achieve a global optimality

Protocol decomposition: TCP/AQM

Duality model: TCP/AQM as distributed primal-dual algorithm over network to maximize aggregate utility (Kelly '98, Low '99, '03)



Protocol decomposition: TCP/AQM

Duality model: TCP/AQM as distributed primal-dual algorithm over the network to maximize aggregate utility (Kelly '98, Low '99, '03)

horizontal decomposition

Optimization and layering



Generalized utility maximization

- Objective function: user application needs and network cost
- Constraints: restrictions on resource allocation (could be physical or economic)
- Variables: Under the control of this design
- Constants: Beyond the control of this design



Layering as optimization decomposition

- Network
- Layers
- Interface
- Layering

Application

IP

Link/MAC

Physical

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- generalized NUM solver sub-problems functions of primal/dual variables decomposition methods
- Vertical decomposition: into functional modules of different layers
- Horizontal decomposition: into distributed computation and control over geographically disparate network components

Layering as optimization decomposition

- Network
- Layers
- Interface
- Layering



- generalized NUM
- sub-problems
- functions of primal/dual variables
- decomposition methods

Provides a top-down approach to design protocol stack

- explicitly tradeoff design objective
- explicitly model constraints and effects of, e.g., new technologies
- provide guidance on how to structure and modularize different functions
- make transparent the interactions among different components and their global behaviors

Optimization and layering

- Yield a much deeper understanding of existing protocols/algorithms and help design better/new ones
 though many unresolved issues remain
- This series of work (starting with Kelly-Low model) had rekindled an interest in theory-based network design
 - several important players: CU (Chen), Caltech (Low, Doyle), Princeton (Chiang), UIUC (Srikant), Purdue/OSU (Shroff, Lin)
 - work on cross-layer design (Chen-Low-Doyle '05, Chen-Low-Chiang-Doyle '06) was among the first
 - see survey articles Chiang et al '07 and Chandra-Gayme-Chen-Doyle '11, and Chen-Low-Doyle '11

Cross-layer design in ad hoc wireless networks

Application	Network performance can be improved if network layers are jointly designed
TCP/AQM	
IP	 Most works design based on intuition, evaluated by simulations unintended consequences
MAC	
Physical	

Cross-layer design in ad hoc wireless networks



Cross-layer design/optimization



Network model

Problem formulation

Network resource allocation:



Protocol decomposition



Cross-layer implementation


Extension to time-varying channel

Application	channel state h : i.i.d. finite state process with distribution $q(h)$
Transport	□ Rate control: → $x(t) = x(p(t)) = \arg \max_{x} \sum_{x} U_{x}(x_{x}) - p^{T}(t)H(x)$
Network	 Routing: solved with rate control or scheduling
Link/MAC	□ Scheduling: → $f(t) = f(p(t)) = \arg \max_{f \in \Pi(h(t))} p^T(t)A(f)$ random
Physical	Congestion update $p(t+1) = \int p(t) d(f(p(t))) H(r(p(t))) dt$
	$p(i+1) = [[p(i) - \gamma_t \{A(f(p(i))) - H(x(p(i)))\}]$

Stability and optimality

<u>Theorem</u> (Chen-Low-Chiang-Doyle '06, '11): The Markov chain is stable. Moreover, the cross-layer algorithm solve the following optimization problem

$$\max_{x,f} \sum_{s} U_{s}(x_{s})$$
s.t. $H(x) \leq A(f)$
 $f \in \overline{\Pi}$
 $\overline{\Pi} = \{\overline{r} : \overline{r} = \sum_{h} q(h)r(h), r(h) \in \Pi(h)\}$

Applicable to any queueing network with interdependent, time-varying, parallel servers

- optimality holds even with time-varying topologies
- throughput-optimal when flow-level dynamics is considered
- A Wi-Fi implementation by Rhee's group at NCSU shows significantly better performance than existing system

Outline

- Layering and constrained optimization (communication network)
- Network dynamics as optimization algorithms (power network)
- Look into future

Smart grid

Power network will go through a similar architectural transformation in the next few decades that telephone network has gone through

deregulation started Tesla: multi-phase AC Enron, blackouts Both started as regulated monopolies 1888 1980-90s 2000s Both provided a single commodity Both were vertically integrated 1876 Both grew rapidly through two WWs 1980-90s deregulation Bell: telephone started

1969:

DARPAnet

- infrastructure dramatically reengineered
- industry landscape drastically reshaped



convergence

to Internet

Emerging trends

- Proliferation of renewable and distributed generation
 The drivers
- Electrification of transportation
- Participation of end users



Deployment of sensing, communication, enablers and computation infrastructure

Proliferation of renewables



real opportunity for sustainability

Sources: REN21, Renewables global status report (2006-2012); DOE/EIA-0383 (2013)

Random/rapid fluctuations



Migration to distributed architecture



2-3x generation efficiency

relieve demand on grid capacity

also in control and management

Denmark's experience

Large-scale network of DERs

Distributed energy resources (DERs): Photovoltaics (PVs), wind turbines, smart loads, inverters, storages, electric vehicles (EVs)



central power plant

Source: ITERES

Large-scale network of DERs

- Challenge: an interconnected system of millions of DERs introducing rapid, large, and random fluctuations in supply, demand, voltage, and frequency
- Opportunity: increased capability to coordinate and optimize their operation optimize their operation optimize their operation of unprecedented efficiency and robustness

Current control paradigm

Hierarchical control structure spanning multiple timescales from subseconds to hours and up



Current control paradigm

- Centralized, open-loop, worst-case preventive, and often human-in-the-loop at slow timescales
 - cope with slow/predictable but often large variations
 - economic efficiency and system security are the key (optimization model)
- Local and automatic at fast timescales
 - cope with fast but relatively small variations
 - □ (local) stability is the key (dynamical model)
 - oblivious of system-wide properties or global perspective
- Sufficient for today's power system
 - relatively low uncertainties, few active assets, mainly to match controllable supply to passive load
 - the lack of ubiquitous sensing, control and communication

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Future control needs

Real-time and close-loop

with rapid, large, and random fluctuations, feedback control based on real-time information is needed

Distributed to ensure scalability

with large number of control points, information must be decentralized and decisions must be made locally

□ Fast/local controls actively and globally coordinated

- Iocal controls must be bridged with the global situation, to ensure system-wide efficiency and robustness
- Enabled by the deployment of sensing, control, and communication infrastructure and the advances in power electronics

New control paradigm

- □ Autonomous DERs for distributed real-time control
 - Each DER made autonomous through local sensing, computing

distributed real-time control with global perspective + layered architecture

what are fundamental challenges?

control as a provide a set of common of common of services to various applications

.av

applications call and synthesize the control services to meet performance specifications

Comparison with the Internet

Partially motivated by the Internet

the precedence on the Internet lends hope to a much larger scale and more dynamic and distributed control architecture

- The physics of electricity cuts through all power system functionalities and operations
 - nonconvexity of power flow
 - dynamics cannot be "designed"

Fundamental challenges

Convexification of power flow

for fast computation for real-time optimization
 for distributed algorithm

Distributed decomposition under dynamics constraints

for distributed real-time control with global perspective
 exploit or implemented as power system dynamics

Integrating sensing, communication, and control

 fundamental limits on control performance arising form sensing constraints and communication constraints
 communication/networking for distributed control

Architecture and layering

mathematical underpinning of smart grid architecture
 systematic methods to develop/evaluate design choices

Fundamental challenges

Convexification of power flow

□ convex relaxation

Distributed decomposition under dynamics constraints

network dynamics as optimization algorithms

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Convexification of OPF

- Optimal power flow (OPF) problem
 - a fundamental problem underlying power system controls and operations
 - huge literature since first formulated in 1962, focusing on approximate algorithms and solutions
- Convexity critical to the development of efficient, distributed, and robust algorithms
 - □ for real-time computation at scale
 - for distributed algorithms
 - for efficient market, as foundation for pricing schemes such as LMP
 - □ for global optimality, required for new/enhanced application

Branch flow model



Power flow constraints

$$V_{i} - V_{j} = z_{ij}I_{ij}$$

$$S_{ij} = V_{i}I_{ij}^{*}$$

$$\sum_{i \to j} \left(S_{ij} - z_{ij} \left|I_{ij}\right|^{2}\right) - \sum_{j \to k} S_{jk} = S_{j}$$

Kirchhoff law power definition power balance

Optimal power flow



OPF:

$$\max \sum_{i} U_{i}(p_{i}) - C\left(\sum_{(0,j)} P_{0j}\right) - r_{i,j} |I_{i,j}|^{2} \quad \text{social welfare}$$

over $x := (P,Q,I,V,p,q)$

s. t. $x \in PFC(x)$ \leftarrow power now constraints $(v_i, p_i, q_i) \in OC(x)$ \leftarrow operation constraints, e.g., in safe range

Convexity structure



OPF:

$$\max \sum_{i} U_{i}(p_{i}) - C\left(\sum_{(0,j)} P_{0j}\right) - r_{i,j} |I_{i,j}|^{2}$$

over $x := (P,Q,I,V,p,q)$
s. t. $x \in PFC(x)$ nonconvex
 $(v_{i}, p_{i}, q_{i}) \in OC(x)$



Convexity relaxation





Exact relaxation

<u>Theorem</u> (Li-Chen-Low '12a): Convex relaxation is exact provided that for any *i*, $v_i^{\text{lin}}(p,q) < \bar{v}_i$ and for any link (*k*,*l*) in the network and (*i*,*j*) on the path from θ to *k*,

If
$$\frac{r_{k,l}}{x_{k,l}} - \frac{X_k}{R_k} > 0$$
, then $v_i + 2\underline{P}_i^{\text{lin}} \left(\frac{r_{k,l}}{x_{k,l}} X_k - R_{i,k} \right) + 2\underline{Q}_i^{\text{lin}} X_i > 0$
Otherwise, $v_i + 2\underline{Q}_i^{\text{lin}} \left(\frac{x_{k,l}}{r_{k,l}} R_k - X_{i,k} \right) + 2\underline{P}_i^{\text{lin}} R_i > 0$

if only load buses, relaxation is always exact
 relaxation is always exact for real systems where

 $v\sim 1, r, x<<1, P, Q<1$

- IEEE distribution test systems
- Southern California Edison circuits
- many decomposition approaches (thus distributed algorithms) apply (Li-Chen-Low '12b, '12c)

Convexification of OPF

Hidden convexity for efficient, distributed computation

- tremendous progress since Lavaei and Low '11; see survey article Low '14
- effectiveness depends on graph properties of underlying physical and/or communication networks
- not always possible, and conditions may violate operation constraints
- □ Convex approximation?
 - geometry of power flow and its dependence on operation constraints and graph properties
 - systematic approach to construct convex approximation, to trade off tractability and optimality

Distributed decomposition under dynamics constraints

Power network is a physical system

- cannot be "re-set" arbitrarily, but has to evolve from one state to another
- algorithms must be "consistent" with system dynamics
- Reverse engineering
 - can we bridge existing local control with system-wide property?
- Forward engineering
 - engineer the model from reverse engineering to guide systematic design of new algorithms



Frequency control



focus on primary control for insight

□ Synchronous generator: $P_i^M = F_i(\omega_i)$

• decreasing; e.g., $F_i(\omega_i) = -S_i\omega_i$

Renewable generator: $P_i^R = H_i(\omega_i)$

deceasing

□ Frequency sensitive load: $P_i^S = G_i(\omega_i)$

• increasing; e.g., $G_i(\omega_i) = D_i\omega_i$

Dynamics

Synchronous generator bus:

$$M_i \dot{\omega}_i = F_i(\omega_i) - G_i(\omega_i) - P_i^I - \sum P_{ij}$$

j:*i*∼*j*

Renewable generator bus:

$$0 = H_i(\omega_i) - G_i(\omega_i) - P_i^I - \sum_{j:i \sim j} P_{ij}$$

no generator):

Load bus (no generator):

$$0 = G_i(\omega_i) + P_i^I + \sum_{j:i \sim j} P_{ij}$$

system state $(\omega(t), P(t))$

Real branch power flow:

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right)$$

Cost/disutility functions

Control functions defines relations between equilibrium frequency and equilibrium generation and load

Synchronous generator: $C_i^M(P_i^M) = -\int_0^{P_i^M} F_i^{-1}(P)dP$

¬ renewable generator: $C_i^P(P_i^R) = -\int_0^{P_i^M} H_i^{-1}(P)dP$

T frequency sensitive load: $C_i^S(P_i^S) = \int_0^{P_i^M} G_i^{-1}(P) dP$

The equivalence of control and decision problem

$$P_i^M = \arg \min_P C_i^M(P) + P\omega_i$$

depend only on the control function but is independent of how the feedback signal is updated

Reverse engineering

<u>Theorem (You-Chen '14a)</u>: Power system dynamics is a distributed primal-dual gradient algorithm to solve

$$\min \sum_{i \in N^{M}} C_{i}^{M} (P_{i}^{M}) + \sum_{i \in N^{R}} C_{i}^{R} (P_{i}^{R}) + \sum_{i \in N} C_{i}^{S} (P_{i}^{S})$$

$$s.t. \qquad P_{i}^{S} + P_{i}^{I} + \sum_{j} P_{ij} = P_{i}^{M}, \quad i \in N^{M} \qquad \text{DC OPF}$$

$$P_{i}^{S} + P_{i}^{I} + \sum_{j} P_{ij} = P_{i}^{R}, \quad i \in N^{R} \qquad \text{problem}$$

$$P_{i}^{S} + P_{i}^{I} + \sum_{j} P_{ij} = 0, \quad i \in N^{L}$$

and the dual variables are frequencies and equal.

network dynamics as optimization algorithms (network as optimizer)

Network dynamics as optimization algorithms

- A new perspective to understand collective behavior arising from interaction between local controls
 - structural properties of the equilibrium point
 - efficiency and tradeoffs, etc
- Suggests a Lyapunov function for global stability or convergence analysis

important both theoretically and practically

Suggests a principled way to systematically design new algorithms and control schemes

Forward engineering

Suggests a principled way to systematically design new algorithms and control schemes

- new design goals (e.g., frequency recovery, fairness, and economic efficiency) incorporated by engineering the global objective function and the constraints
- new control schemes with different dynamical properties and complexities based on various optimization algorithms
- insights from reverse engineering can guide particular way to engineer the model and derive the algorithm

Nominal frequency recovery

■ Key observation: $\omega = 0$ can be ensured if $\sum_{i \in N} G_i(\omega) = 0$ at equilibrium

 $\sum_{i \in N} P_i^I = \sum_{i \in N^M} P_i^M + \sum_{i \in N^R} P_i^R \quad \begin{array}{c} \text{do not have the} \\ \text{decoupling structure} \end{array}$

Impose the above indirectly by imposing decoupling constraints

$$P_i^I + \sum_{j} Q_{ij} = P_i^M, \quad i \in N^M$$
$$P_i^I + \sum_{j} Q_{ij} = P_i^R, \quad i \in N^R$$
$$P_i^I + \sum_{j} Q_{ij} = 0, \quad i \in N^L$$
$$Q_{ij} = Q_{ji}$$

Nominal frequency recovery

A new optimization problem (You-Chen '14a):

$$\max \sum_{i \in N^{M}} C_{i}^{M} (P_{i}^{M}) + \sum_{i \in N^{R}} C_{i}^{R} (P_{i}^{R}) + \sum_{i \in N} C_{i}^{S} (P_{i}^{S})$$

$$s.t. \qquad P_{i}^{S} + P_{i}^{I} + \sum_{j} P_{ij} = P_{i}^{M}, \quad i \in N^{M}$$

$$P_{i}^{S} + P_{i}^{I} + \sum_{j} P_{ij} = P_{i}^{R}, \quad i \in N^{R}$$

$$P_{i}^{S} + P_{i}^{I} + \sum_{j} P_{ij} = 0, \quad i \in N^{L}$$

$$P_{i}^{I} + \sum_{j} Q_{ij} = P_{i}^{M}, \quad i \in N^{M}$$

$$P_{i}^{I} + \sum_{j} Q_{ij} = P_{i}^{R}, \quad i \in N^{R}$$

$$P_{i}^{I} + \sum_{j} Q_{ij} = P_{i}^{R}, \quad i \in N^{R}$$

$$P_{i}^{I} + \sum_{j} Q_{ij} = 0, \quad i \in N^{L}$$

Nominal frequency recovery

New control scheme (You-Chen '14a):

$$P_i^M = F_i(\omega_i), \ i \in N^M$$

$$P_i^R = H_i(\omega_i + \mu_i), \ i \in N^R$$

$$\dot{v}_i = -(G_i(\omega_i) + \sum_j (P_{ij} - Q_{ij}))/M_i, \ i \in N^M$$

$$\dot{\mu}_i = \xi_i (G_i(\omega_i) + \sum_j (P_{ij} - Q_{ij})), \ i \in N^R \cup N^L$$

$$\dot{Q}_{ij} = \varepsilon_{ij} (\mu_i - \mu_j)$$

distributed control
Economic dispatch

<u>Theorem</u> (You-Chen '14a): Power system dynamics with the new control scheme solves economic dispatch problem

$$\min \sum_{i \in N^{M}} C_{i}^{M} (P_{i}^{M}) + \sum_{i \in N^{R}} C_{i}^{R} (P_{i}^{R}) + \sum_{i \in N} C_{i}^{S} (P_{i}^{S})$$

$$s.t. \qquad P_{i}^{I} + \sum_{j} P_{ij} = P_{i}^{M}, \quad i \in N^{M}$$

$$P_{i}^{I} + \sum_{j} P_{ij} = P_{i}^{R}, \quad i \in N^{R}$$

$$P_{i}^{I} + \sum_{j} P_{ij} = 0, \quad i \in N^{L}$$

- real-time frequency control recovering frequency and achieving economic efficiency at the same time
- different from current approach achieving these objectives at different timescales and with centralized control
- needed for future smart grid to cope with rapid/large fluctuations and manage a huge number of control points

Network dynamics as optimization algorithms

Natural system dynamics exploited for simplicity, scalability, and robustness

desired for distributed real-time control

Lots progress

Iocal volt/var control (Farivar-Chen-Low '13); automatic generation control (Li-Chen-Zhao-Low '14); load side frequency control (Zhao *et al* '12, '14, Mallada *et al* '14); distributed frequency control in microgrids (Dorfler *et al* '14)

More work needed

- remove approximations
- integrate frequency and voltage control
- distributed decomposition of AC OPF problem

Outline

- Layering and constrained optimization (communication network)
- Network dynamics as optimization algorithms (power network)
- Look into the future

Layering and optimization

Architecture (foundations and design methodologies)

networks and protocols & dynamics as distributed decomposition of optimization

Derive the layering structure and modularity of various mechanisms

Make transparent the interactions among different components and their global behaviors

Mathematical underpinning of network architecture

Architecture (foundations and design methodologies)

networks and protocols & dynamics as distributed decomposition of optimization

- A common analytical framework and language
 - handle and integrate sensing, computation, communication, control, and incentives
 - allow rigorous analysis and systematic design
- Close the gap between theory and practice
 - implementation and verification
 - theory and implementation inform each other

Approach



must be both abstract and concrete must be both foundational and practical

SDN controller architecture

Software-defined networking (SDN)

- decouple control and data planes
- dynamic and active networking
- network virtualization and controller as network OS
- Aim to develop models/theories to guide the analysis and design
 - distributed controller architecture
 - dynamic resource (re-)allocation and management algorithms



SDN architecture (open networking foundation)

Energy-efficient data center

- A new branch of research with its own rich structures and unique challenges
- Aim to develop models/theories to guide the analysis and design of practical algorithms for energy efficient data centers
 - our initial step on some of these issues (Chen-Li-Low '10, Chen-Li '13, Chen-Andrew-Wierman '14)
 - Iots of work existed already



3-5% of total US energy use

Smart grid





- Nonconvexity of power flows
 - convex approximation
- Network dynamics as optimization algorithms
 - distributed decomposition of AC OFP
- Integrating sensing, communication, control
 - fundamental limits on control performance under sensing/communication constraints (You-Chen '14b, Shihadeh-You-Chen '14)
 - communication/networking for distributed control
 - Architecture and layering
 - mathematical underpinning & systematic methods
 - framework to reason about architectural questions
 - design goals
 - design principles: layering, division of functionality, placement of intelligence, ...

Research agenda

communication networks

core Theory challenges (comm., comp., contr., sensing; optimization, game systems,)

power networks learning, inference, sparse sampling, parsimonious solutions

Thank you!

Electricity: 1800...



... (most) architecture today

Electricity: today...

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2} \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{A}$$

... and our goal here

methodological transformation