Optimization and Control of Networks

# Random Access Games and Medium Access Control Design

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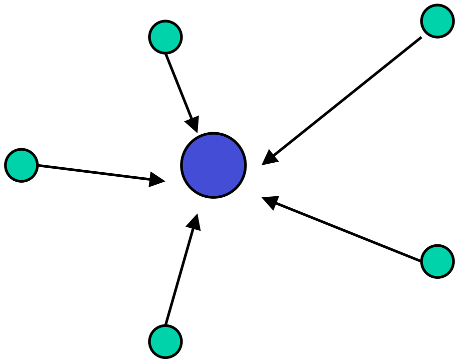
# Agenda

- Contention-based medium access control (contention control)

- A game theoretic approach to contention control

# Medium access control (MAC)

- Wireless channel is shared medium and interference-limited
- Medium access control: coordinate channel access
  - Reduce/avoid interference/collision
  - Efficient utilization of wireless spectrum
  - Quality of Service control



a multiple access network

# Two kinds of methods

- Schedule-based
  - Establish transmission schedules *a priori* or dynamically
  - Usually requires centralized implementation
  - High complexity, not practical in real networks
- Contention-based
  - Wireless nodes contend for the channel
  - Simple, distributed implementation
  - High statistical multiplexing gain
  - Aloha, CSMA/CA, 802.11 DCF, …

# Aloha

- Very simple: if a node has a packet to send, it just transmits
- Listen for an amount of time
  - If an ACK is received, done.
  - Otherwise, resend the packet
- Low-delay in light-load scenarios
- Low channel utilization (<=18%)
  - Collision window is equal to transmission time (TT) plus propagation delay (PD)

# Slotted Aloha

- Time is slotted
  - slot duration is equal to transmission time plus maximum propagation delay
- Begin transmission at the slot boundaries
- Higher channel utilization (<=1/e)
  - Collision window is a point -- the slot boundary

# Carrier Sensing multiple access (CAMA)

- Infer channel state through carrier sensing
  - Sense carrier before transmission
  - If idle, transmit the whole packet
  - Wait for ACK
- Higher channel utilization
  - Collision window is equal to maximum propagation delay
- When finding a busy channel
  - Non-persistent: sense the channel again after a random amount of time; if idle, send immediately
  - P-persistent: sense continuously; if idle, send with probability p

# Contention/collision resolution

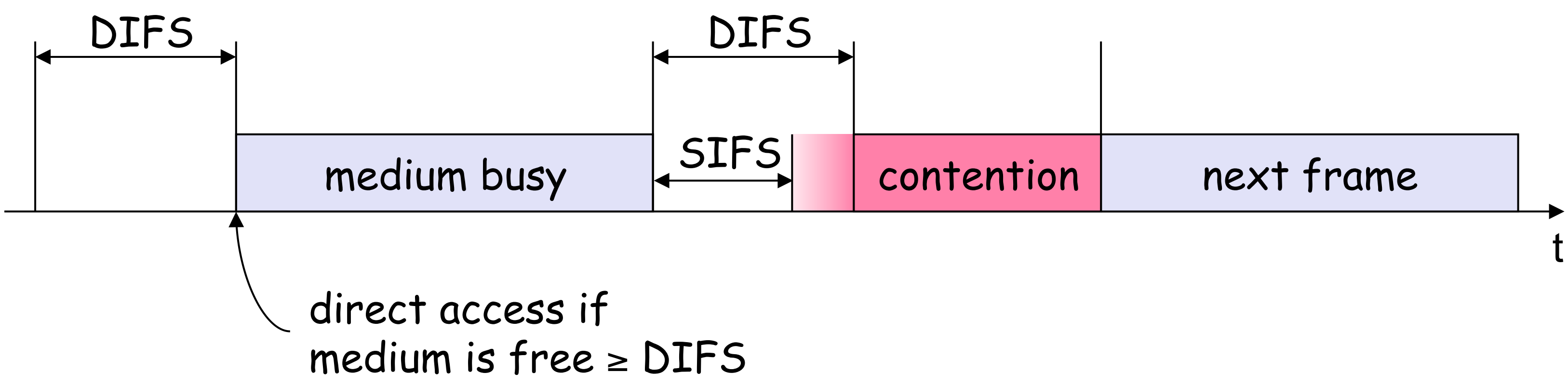
- What to do upon a collision
  - If the colliding nodes transmit immediately when the channel is idle after a collision, another collision is guaranteed
- Two collision resolution mechanisms
  - Persistence: transmit with a probability p
  - Backoff: wait for a random amount of time bounded by CW before retransmission
- Contention resolution algorithm (i.e., how to decide p and CW values dynamically in response to contention) is the key

# Wireless 802.11 DCF (basic)

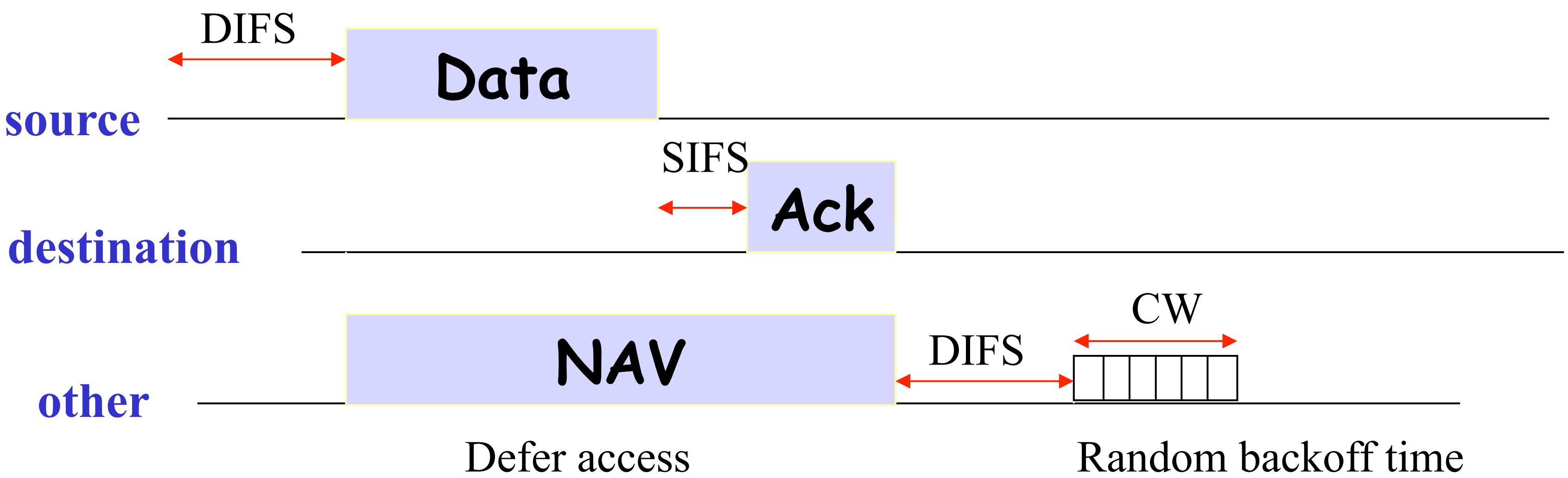
- DCF stands for distributed coordination function
- A CSMA/CA medium access protocol
  - CSMA: sense before transmission
  - CA: random backoff to reduce collision probability
    - when transmitting a packet, choose a backoff interval in the range [0, CW-1]
  - Count down the backoff interval when medium is idle
    - count-down is suspended if medium becomes busy
  - Transmit when backoff interval reaches 0

- Contention resolution: contention window CW is adapted dynamically depending on collision occurrence
  - binary exponential backoff: double CW upon every collision
  - Set to base value (CW=32) after a successful transmission
  - Packet collision is the feedback signal

- Slotted system: Inter Frame Spacing
  - SIFS (Short Inter Frame Spacing)
    - highest priority, for ACK, CTS
  - DIFS (Distributed Coordination Function IFS)
    - lowest priority, for asynchronous data service



# DCF basic access method



# Agenda

- Contention-based medium access control (contention control)

- A game theoretic approach to contention control

# Contention-based MAC (contention control)

- Medium access control (MAC): coordinate channel access
  - avoid collision
  - efficient utilization of wireless spectrum
  - Quality of Service control
- Contention resolution mechanisms
  - persistence: transmit with a probability p
  - backoff: wait a random amount of time bounded by contention window CW before transmission
- Contention resolution algorithm is the key
  - i.e., decide p or CW value in response to network contention

# Wireless 802.11 distributed coordination function (DCF)

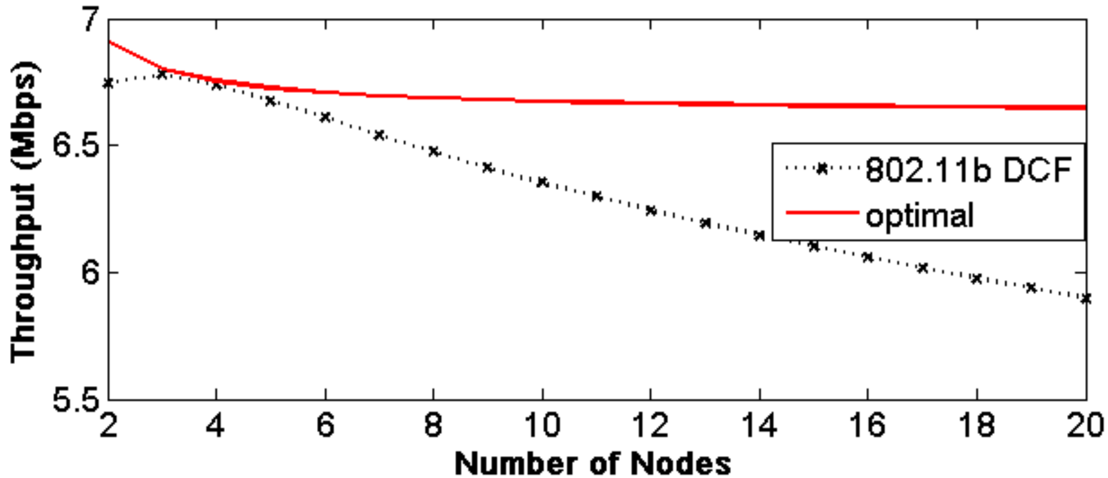
- Contention resolution algorithm: Binary exponential backoff

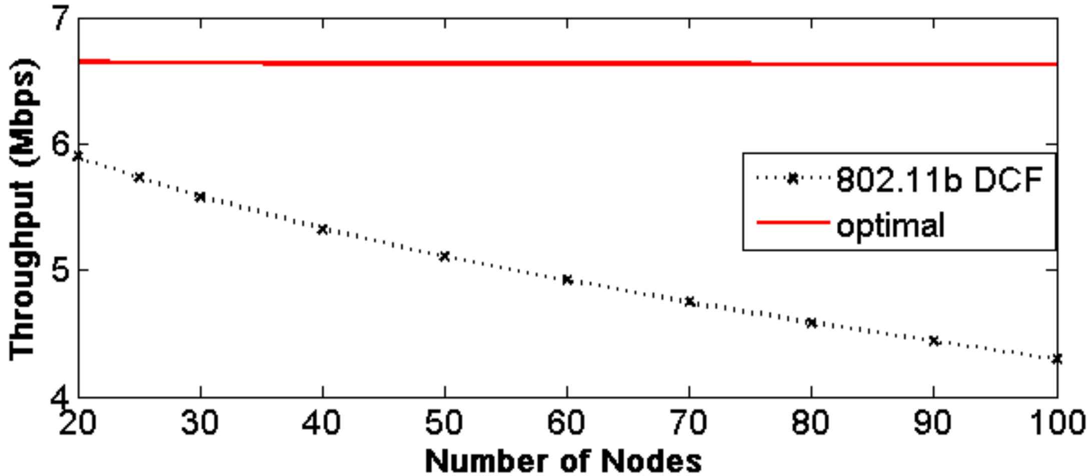
$$CW \leftarrow 2CW, \qquad if\,(\text{collision})$$

$$CW \leftarrow CW_0, \quad if\,(\text{successful transmission})$$

  - respond to a binary feedback signal - packet collision
- Performance problems
  - excessive collision and low throughput
  - poor short-term fairness
  - cannot distinguish packet collision from corrupted frame

# DCF throughput

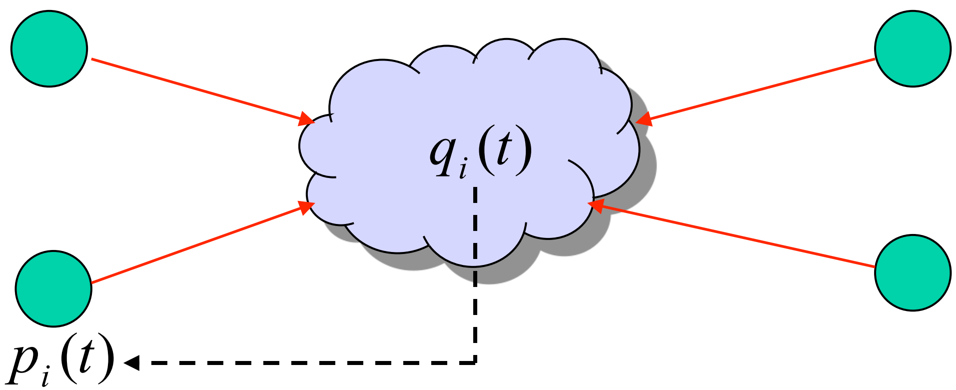


# Better design

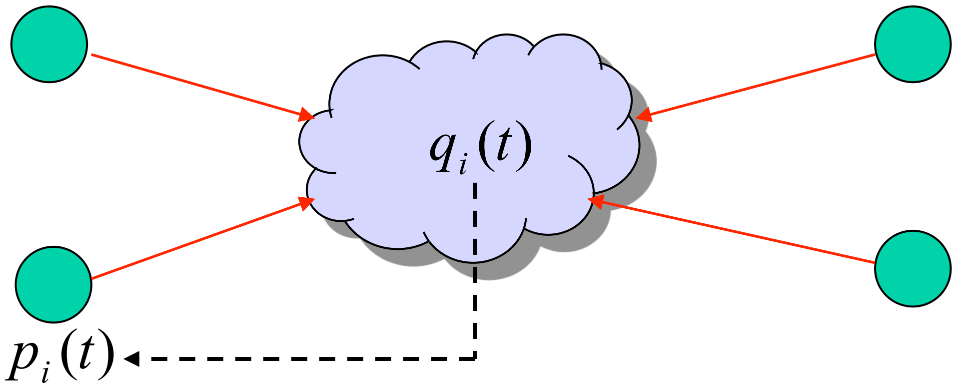
- Many works exist
  - mostly based on intuition and heuristics and evaluated by simulation
  - optimal design, but with sophisticated methods to estimate the number of contending nodes
- Our “theory-based” approach
  - reverse engineering: see what mathematical problem contention control implicitly solves
  - forward engineering: understand and engineer the underlying problem to derive the design in a formal and structured way

# Contention control: dynamical model




- Two components
  - contention resolution algorithm: adjusts channel access probability in response to contention
    - e.g., DCF uses binary exponential backoff
  - feedback mechanism: updates a contention measure and sends it back to wireless nodes
    - e.g., DCF uses a binary contention measure - packet collision

# Contention control: dynamical model




- Dynamical model

$$p_i(t+1) = \mathcal{F}_i(p_i(t), q_i(t))$$

$$q_i(t+1) = \mathcal{G}_i(p(t))$$

  - the exact form of $\mathcal{F}_i$ and $\mathcal{G}_i$ are determined by or can be designed for the specific MAC protocol

- Present a game-theoretic model to understand the above dynamical system and use it to design new protocols

# Random access game

$$p_i(t+1) = \mathcal{F}_i(p_i(t), q_i(t))$$
$$q_i(t+1) = \mathcal{G}_i(p(t))$$

$$\Big\downarrow \text{ fixed point}$$

$$\boxed{p_i = \mathcal{F}_i(p_i, q_i)}$$

$$\Downarrow$$

$$\boxed{q_i = F_i(p_i)}$$

$$\Downarrow$$

$$\boxed{U_i(p_i) = \int F_i(p_i)dp_i}$$

- determined by the contention resolution algorithm
- usually continuous, increasing, and concave

# Random access game

**Definition** (Chen el al '06; '10): A random access game is defined as a quadruple

$$\mathbf{G} := \{N, (S_i)_{i \in N}, (u_i)_{i \in N}, (q_i)_{i \in N}\}$$

- $N$ is a set of players (wireless nodes)
- strategy $S_i := \{p_i \mid p_i \in [v_i, w_i]\}$ with $0 \le v_i \le w_i < 1$
- payoff function $u_i(p) := U_i(p_i) - p_i q_i(p)$ with certain contention measure $q_i = \mathcal{G}_i(p)$

# Random access game

Contention control can be seen as a distributed strategy update algorithm solving the random access game

- the steady state properties can be understood and designed through the specification of $U_i$ and $q_i$
  - conditional collision probability $q_i(p) = 1 - \prod_{j \neq I_i} (1 - p_j)$ as contention measure
- the adaptation of channel access probability can be specified through $(\mathcal{F}_i, \mathcal{G}_j)$, corresponding to different strategies to approach the equilibrium

- Conditional collision probability as contention measure

$$q_i(p) = 1 - \prod_{j \neq I_i} (1 - p_j)$$

- Assumptions (single cell wireless LANs):
  - A0: $U_i(\cdot)$ is continuously differentiable, strictly concave, and with bounded curvature away from zero, i.e.,

    $$1/\mu \geq -1/U_i''(p_i) \geq 1/\lambda > 0$$

  - A1: let $\gamma(p) = \prod_i (1 - p_i)$ and denote the smallest eigenvalue of $\nabla^2 \gamma(p)$ by $\nu_{\min}$. Then, $\mu + \nu_{\min} > 0$.
  - A2: functions $\Gamma_i(p_i) = (1 - p_i)(1 - U_i'(p_i))$ are all strictly increasing or all strictly decreasing

# Equilibrium

**Theorem**: The random access game has a unique Nash equilibrium (NE).

- a channel access probability $p^*$ is a Nash equilibrium of random access game, if

$$u_i(p_i^*, p_{-i}^*) \geq u_i(p_i, p_{-1}^*),\ \forall p_i \in S_i,\ \forall i \in N.$$

- Proof: Equilibrium condition

$$(U_i^{'}(p_i^*) - q_i(p^*))(p_i - p_i^*) \leq 0, \forall p_i \in S_i$$

is optimality condition for a strictly convex optimization

$$\max_p \sum_i (U_i(p_i) - p_i) - \prod_i (1 - p_i)$$

# Symmetric equilibrium

**Definition**: A NE $p^*$ is said to be symmetric if $p_i^* = p_j^*$ for wireless nodes $i, j$ in the same class, and an asymmetric equilibrium otherwise.

**Theorem** (CLD '06; CLD '10): The random access game has a unique and symmetric NE.

Implications:

- guarantees fair sharing of wireless channel among the same class of wireless nodes
- provides service differentiation among different classes of wireless nodes

# Dynamics (learning algorithms)

- Studies how interacting players (wireless node) could converge to a NE
- In setting of random access
  - players (wireless nodes) can observe outcome of others' actions (i.e., to sense the carrier)
  - players do not have direct knowledge of other players' actions or payoffs
- Consider repeated play of the random access game, and look for distributed strategy update mechanism to achieve NE

# Gradient play

$$p_i(t+1) = [p_i(t) + \varepsilon_i (U_i^{'}(p_i(t)) - q_i(p(t)))]^{s_i}$$

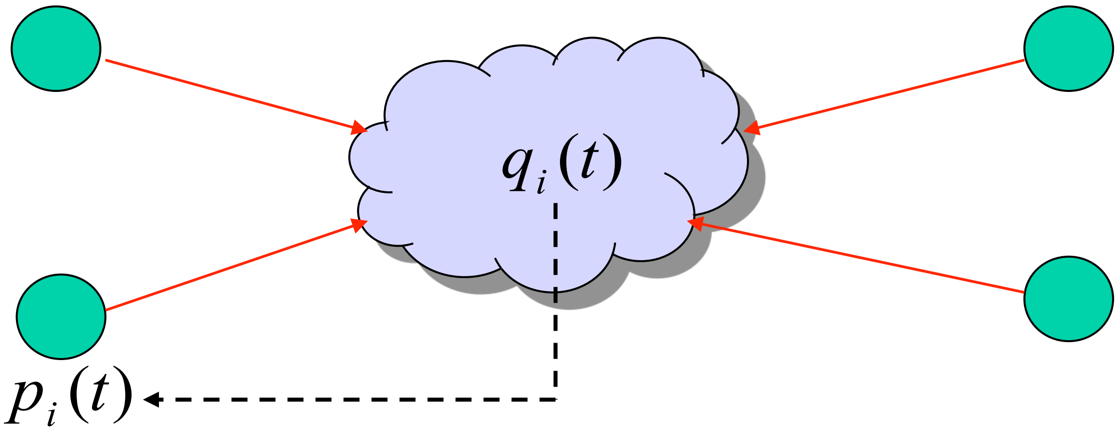
<u>Theorem</u> (CLD '06; CLD '10): The gradient play converges to the unique NE if stepsize $\varepsilon_i < \frac{2}{\lambda + |N| - 1}$. for any $i \in N$.

- proof by Lyapunov method.
- also studied its robust verification to estimation error (CLD '10)
- extensions to multi-cell networks (CLD '10)

# MAC design

- Design MAC according to distributed strategy update algorithm to achieve the equilibrium of random access game
  - by appropriately choosing utility function and contention measure, we can achieve different performance objectives
  - can choose to implement different converging algorithms to the same equilibrium
    - same equilibrium property but different dynamical properties

# Medium access method via gradient play




each wireless node estimates its conditional collision probability and updates its channel access probability according to the gradient play

- by appropriately choosing utility functions, we can achieve different performance objectives
- conditional collision probability can be estimated by sensing idle periods

# Medium access method via gradient play

```
After each transmission
{
 /* Wireless node observes n idle slots before a transmission*/
 isum ← isum + n
 ntrans ← ntrans + 1
 if ( ntrans ≥ maxtrans ){
    /*compute the estimator*/
    n̄ ← βn̄ + (1 − β)isum / ntrans
    q_i ← (1 − (n̄ + 1)p_i)/((n̄ + 1)(1 − p_i))
    /*update access probability*/
      p_i ← p_i + ε_i(U'_i(p_i) − q_i)
    /*update contention window*/
    cw_i ← (2 − p_i)/ p_i
    /*reset variables*/
    isum ← 0
    ntrans ← 0
    }
}
```

- Adapt to continuous feedback signal
- Equation-based control

# A concrete MAC design

- Consider a single-cell network with $L$ classes of users
- Each class $l$ associated with a weight $\phi_l$
- Want to achieve maximal throughput under the weighted fairness constraint

$$\frac{T_l}{T_m} = \frac{\phi_l}{\phi_m}, 1 \le l, m \le L.$$

# Utility design

- Let $\xi = \sum_i p_i$, under the assumption of Poisson arrival, the throughput achieves maximum at $\xi^*$ that satisfies

$$(1-\xi^*)e^{\xi^*} = 1-\sigma/\mathrm{T}_c$$

  - $\sigma$ the duration of idle slot, $\mathrm{T}_c$ the duration of a collision
- Under the decoupling approximation, to achieve weighted fairness requires

$$\frac{p_l}{p_m} = \frac{\phi_l}{\phi_m}, 1 \le l, m \le L.$$

- Utility function

$$U_l(p_l) = (1+\frac{e^{-\xi^*}}{\phi_l})p_l + e^{-\xi^*}(1+\frac{1}{\phi_l})\ln(1-p_l)$$

$$p_l \in [0, w]$$

# Equilibrium and dynamics

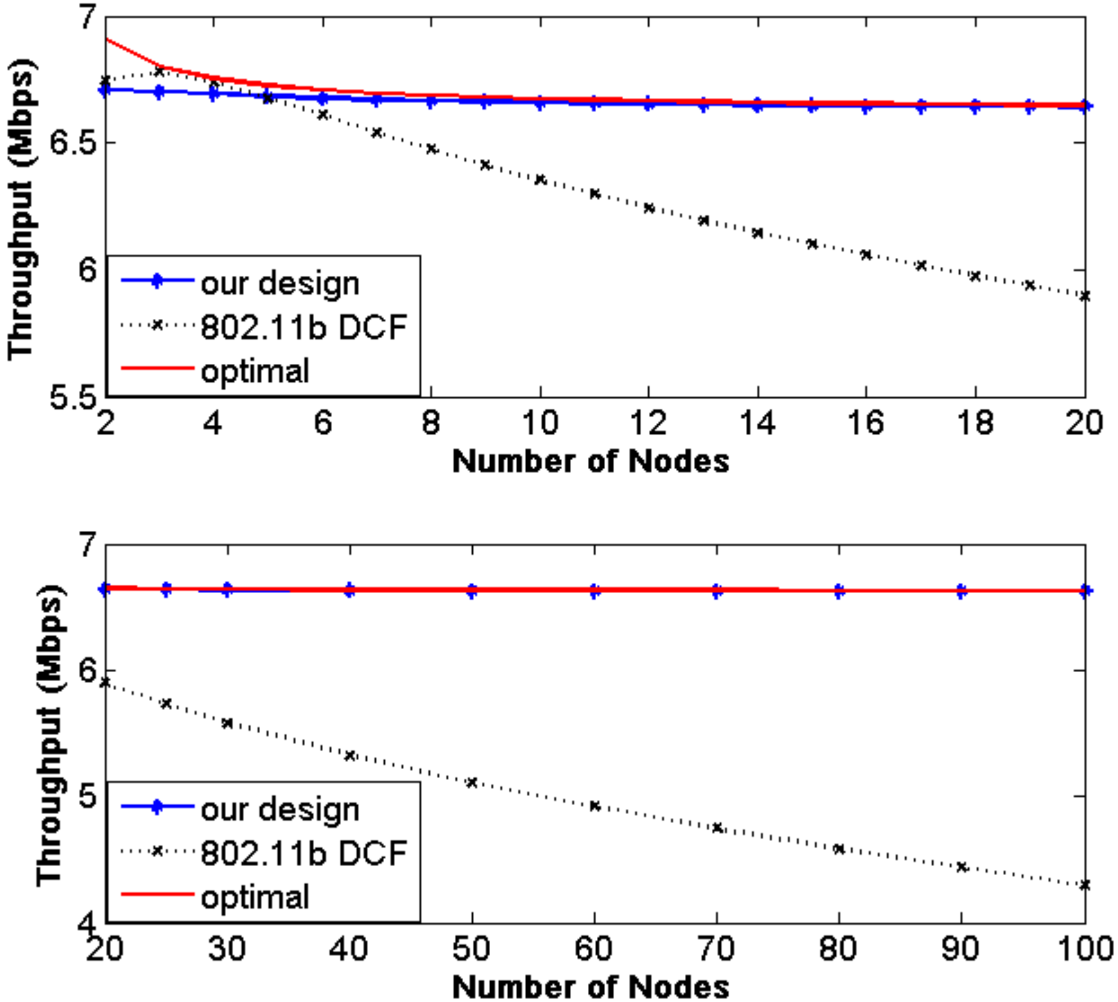
**Theorem** (Chen el al ' 10): Suppose

$$\frac{1-e^{-\xi}}{1+e^{-\xi}/\phi_{\max}} \le w < 1-\frac{e^{\xi}}{1+1/\phi_{\max}}.$$

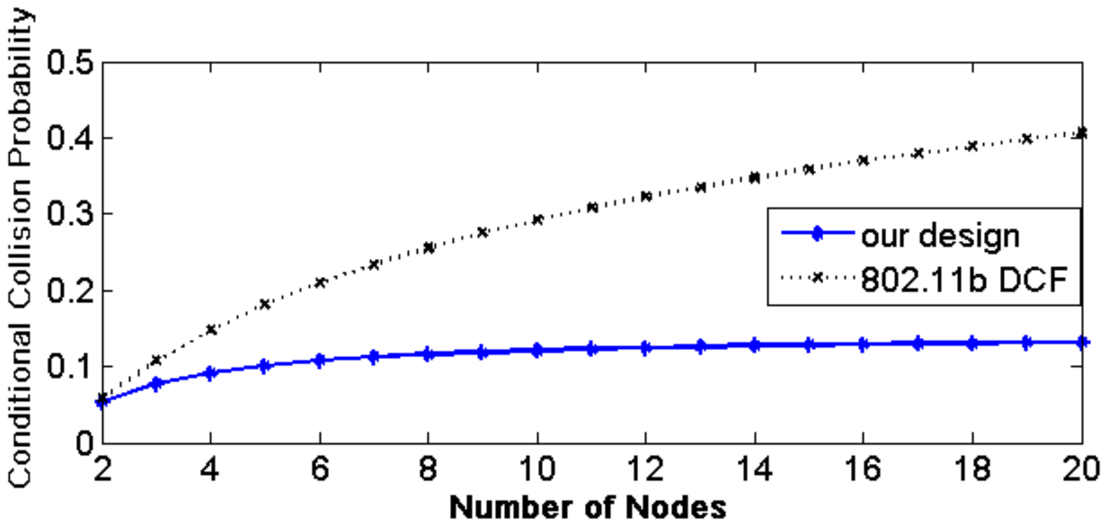
The random access game has a unique and symmetric NE, and the gradient play converges.

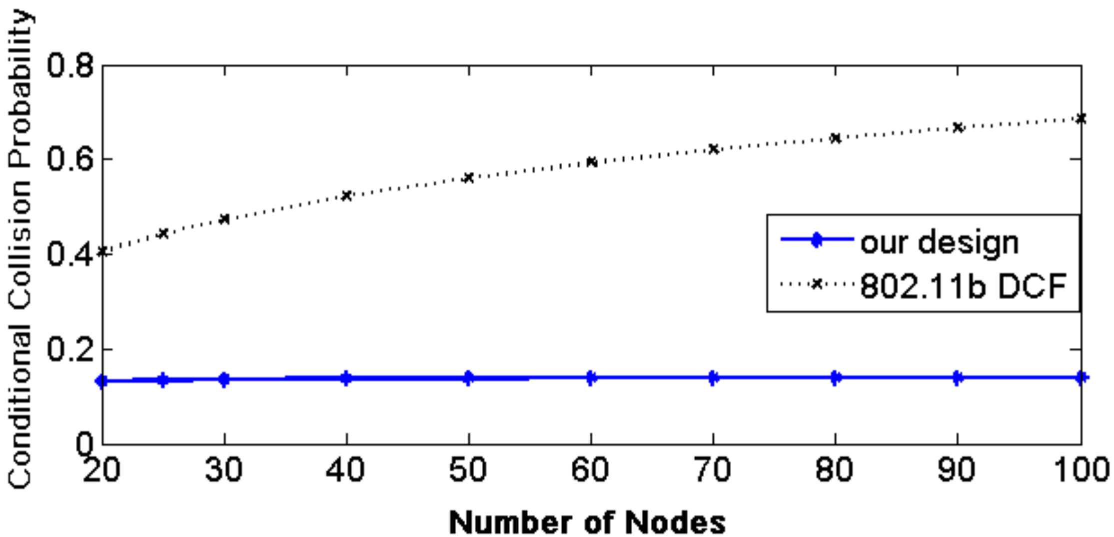
- allows a very large design space

# Performance: throughput

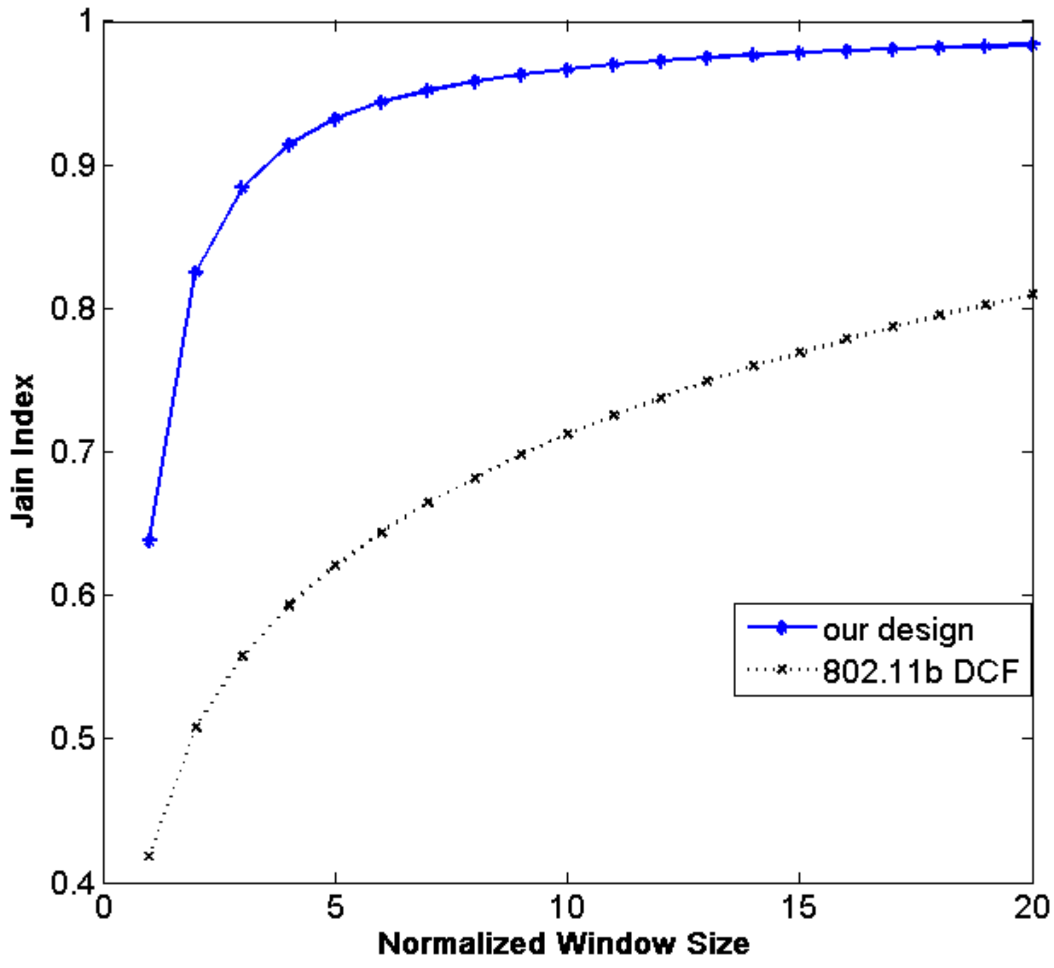


# Performance: collision

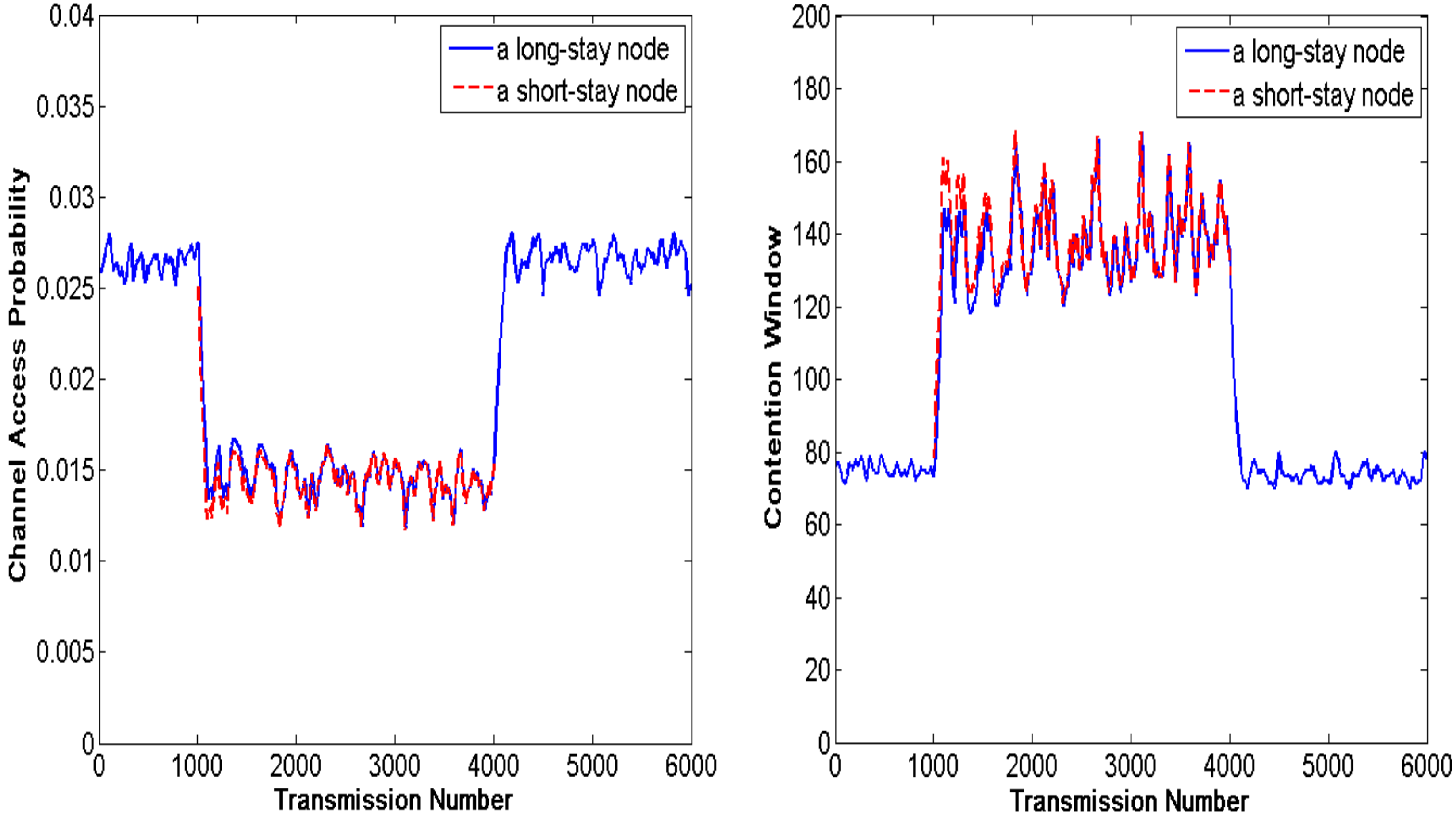


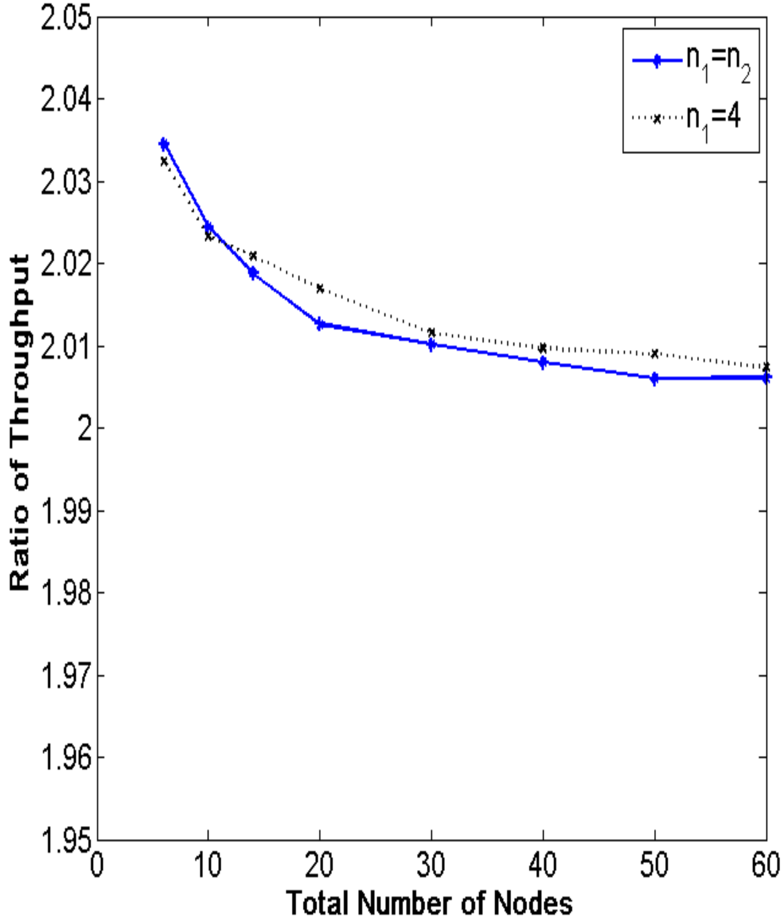
# Performance: short-term fairness

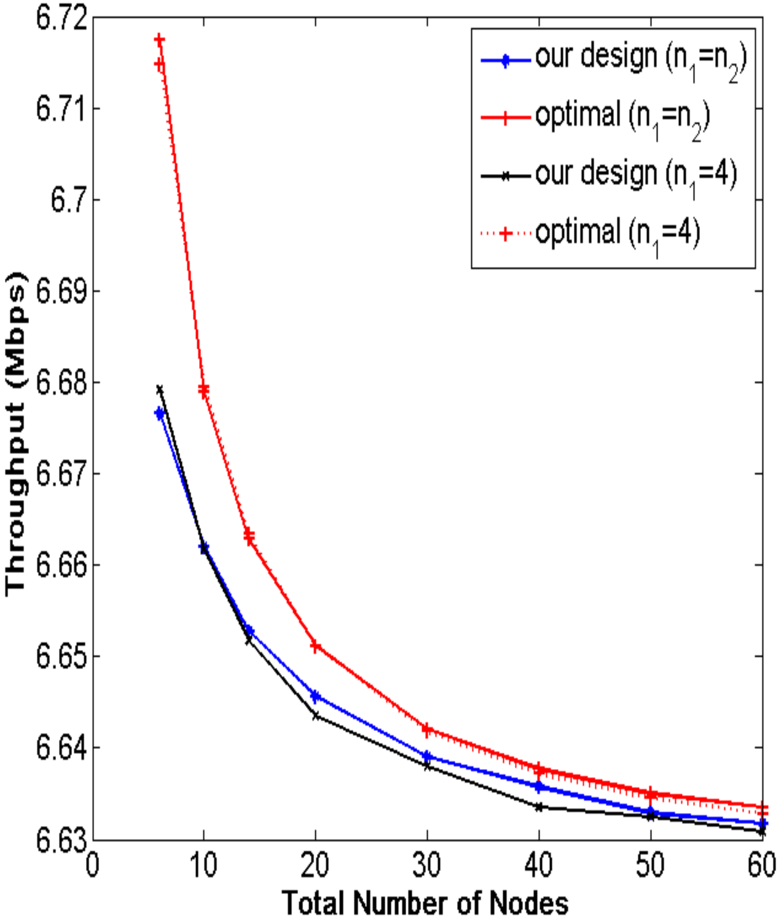


# Performance: dynamic scenario



# Performance: service differentiation



# Game theory based decomposition

system-wide performance objective

↓

design agent utility
and define game

↓

look for distributed
converging algorithm

↓

protocol design:
distributed strategy update algorithm
implemented as network protocol