Exercise 5 Refinery scheduling problem

A refinery has one input, crude oil (CO), and produces 3 outputs:

Gasoline
Diesel
Kerosene

The technology is call a Constant Elasticity of Transformation (CET) function, producing multiple outputs from one input.

$$\left(\sum_i \alpha_i \left(\frac{X_i}{\alpha_i} \right)^{\beta} \right)^{\frac{1}{\beta}} = CO \qquad \infty \geq \beta \geq 1, \qquad \sigma = \frac{1}{\beta - 1}$$

Note that if β = 2, for example, this is just the equation of a circle. This special case has an elasticity of transformation σ = 1.

If there are only two outputs, we would simply call this the PPF.

Let p_i denote the price of product i. The "unit revenue function" is a value function, the *maximum* revenue from one unit of input.

$$r(p) = \underset{x}{Max} \sum p_i x_i + \lambda \left[\left(\sum_i \alpha_i \left(\frac{x_i}{\alpha_i} \right)^{\beta} \right)^{\frac{1}{\beta}} - 1 \right]$$

If you do the algebra, this unit value function is given by

$$r(p) = \left(\sum \alpha_i p_i^{\sigma + 1} \right)^{\frac{1}{\sigma + 1}} \qquad R(p) = \left(\sum \alpha_i p_i^{\sigma + 1} \right)^{\frac{1}{\sigma + 1}} CO$$

where *R(p)* is the total revenue derived from *CO* units of input. Applying Shepard's lemma to *R(p)*, optimal outputs are

$$\frac{\partial R(p)}{\partial p_i} = X_i = ????$$

Exercise 5

(A) given the revenue function, apply Shepard's lemma to get the optimal supply functions for the three products

(B) solve for optimal product outputs using exercise-q5.gms

- NLP formulation
- MCP formulation

$$\frac{\partial R(p)}{\partial p_i} = X_i = \alpha_i p_i^{\sigma} r(p)^{-\sigma} CO$$