Slides for Chapter 4: Examples of familiar industrial-organization problems modeled in GAMS

4.1 Cournot and Bertrand oligopoly with continuous strategies Application to strategic trade policy

Two firms h and f (as in countries h and f) produce imperfect substitutes for the world market:

(a) linear inverse demand curve for each good

(b) each firm has a constant marginal cost

(c) fixed costs are ignored.

$$p_h = \alpha - \beta X_h - \gamma X_f \qquad p_f = \alpha - \beta X_f - \gamma X_h \qquad \beta \geq \gamma$$

$$\pi_i = p_h X_i - c_h X_i = (\alpha - \beta X_i - \gamma X_j) X_i - c_h X_i \qquad i \neq j$$

Best response Cournot-Nash equilibrium is the solution to the two first-order conditions for h and f

$$c_h \geq \alpha - 2\beta X_h - \gamma X_f \qquad c_f \geq \alpha - 2\beta X_f - \gamma X_h$$

These FOC are commonly referred to as “best response” or “reaction” functions.

They can be solved explicitly, easy in symmetric case with identical marginal costs (the two outputs are then identical):

$$X_i = \frac{\alpha - c}{2\beta + \gamma}$$

Figure 20.1

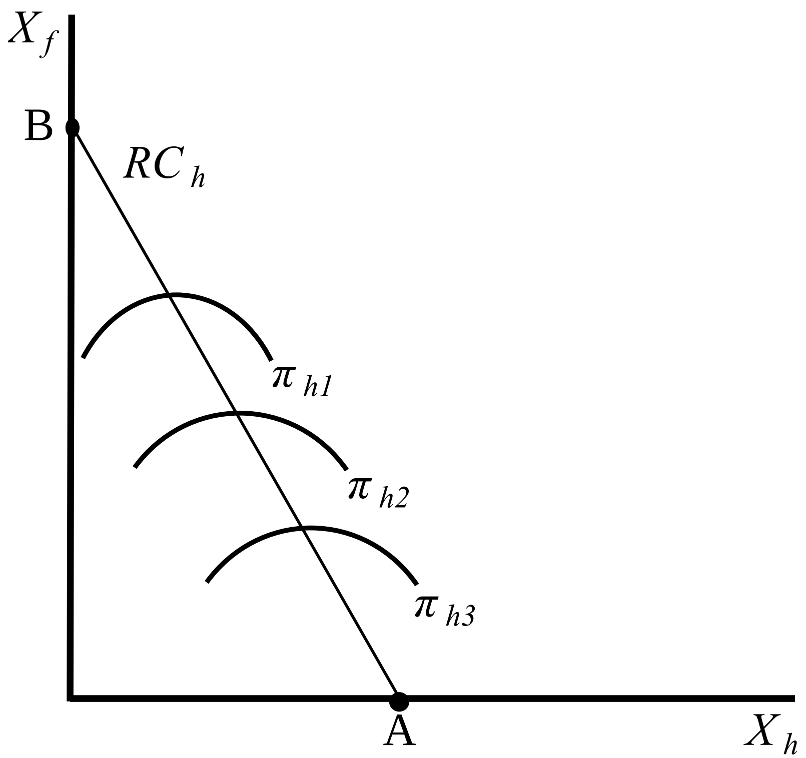
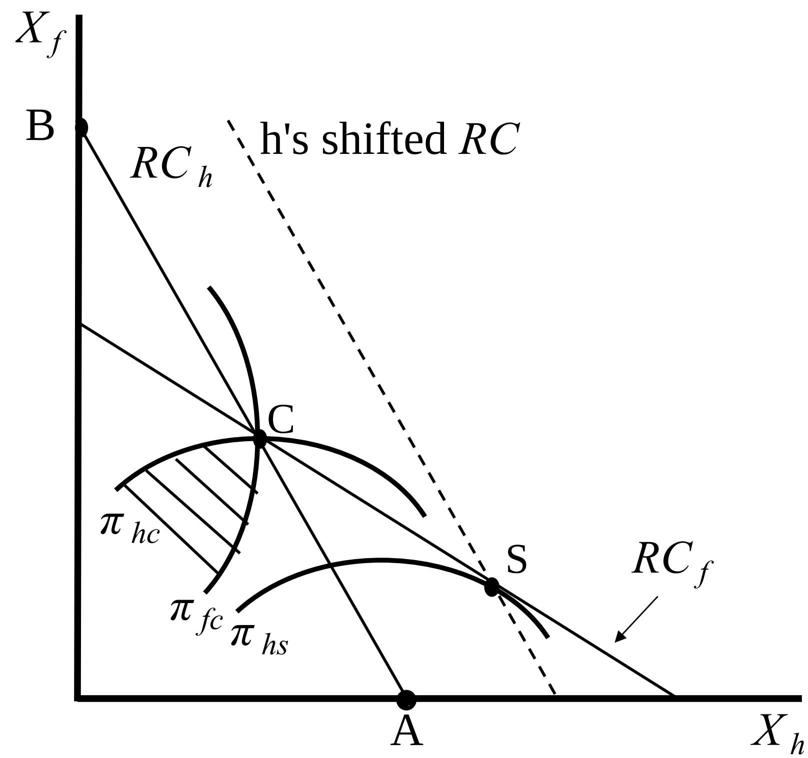


Figure 20.2



Bertrand: best response Nash with prices as the strategic variable requires strict inequality $\beta > \gamma$

Must invert the inverse demand functions:

$$X_i = \alpha_b - \beta_b p_h + \gamma_b p_f$$

$$\alpha_b = (\alpha * \beta - \alpha * \gamma)/(\beta^2 - \gamma^2)$$

$$\beta_b = \beta/(\beta^2 - \gamma^2) \qquad \gamma_b = \gamma/(\beta^2 - \gamma^2)$$

Profits for firm i are:

$$\pi_i = (p_i - c_i)X_i = (p_i - c_i)(\alpha_b - \beta_b p_i + \gamma_b p_j)$$

Best response Bertrand-Nash equilibrium is the solution to the two first-order conditions for h and f

$$-\beta_b c_h \geq \alpha_b - 2\beta_b p_h + \gamma p_f \qquad \beta_b c_f \geq \alpha_b - 2\beta_b p_f + \gamma_b p_h$$

These are the “best response” or “reaction” functions for Bertrand, and are easy to solve in the symmetric case:

$$p_i = \frac{\alpha_b + \beta_b c}{2\beta_b - \gamma_b}$$

Reaction function are

negatively sloped for Cournot (strategic substitutes)

positively sloped for Bertrand (strategic complements).

Figure 20.3

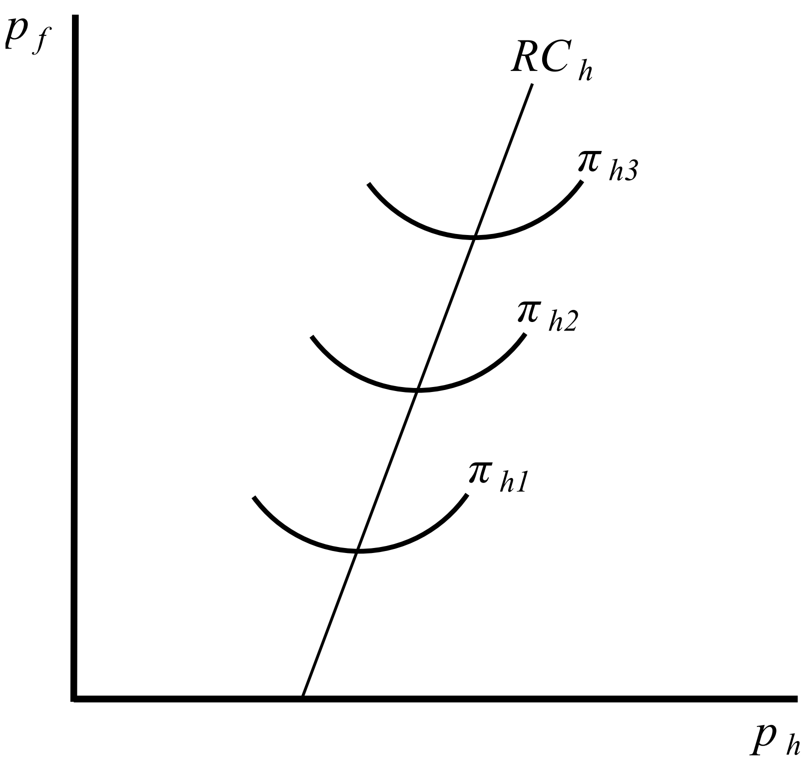
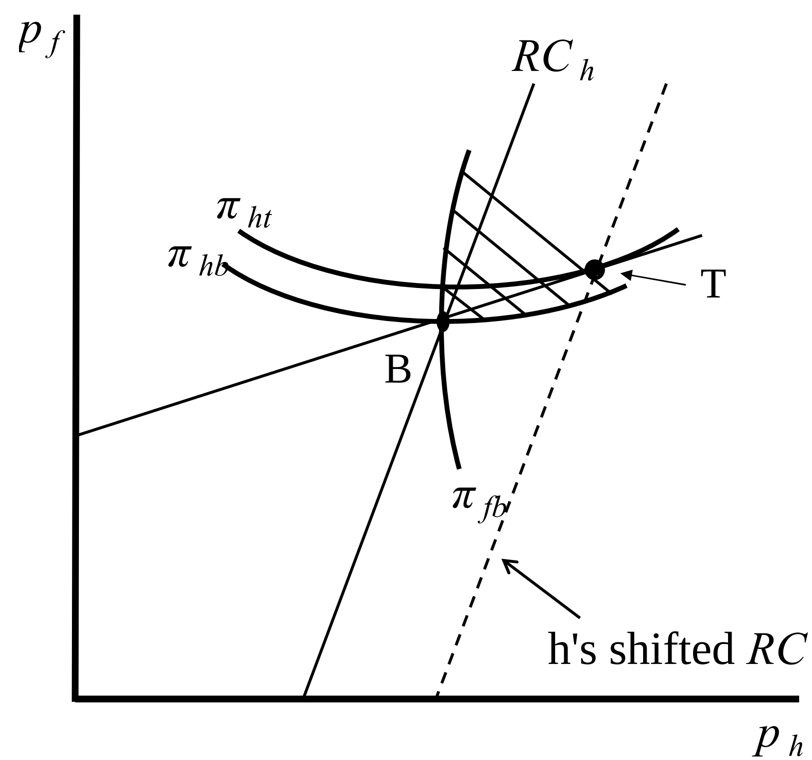


Figure 20.4



“Strategic trade policy”:

Can a government increase welfare by shifting market share and profits to its domestic firm through a subsidy s?

Assume all output sold in a third country. Then welfare of country i is just the profits of its firm minus subsidy payments.

$$WELF_i = \pi_i - s_i X_i$$

Cournot: “optimal” non-cooperative policy is s > 0 (positive subsidy)

Bertrand: “optimal” non-cooperative policy is s < 0 (positive tax)

Can show: bilateral non-cooperative (Cournot) policy is subsidies, cooperative policy is bilateral taxes

```
$TITLE: M4-1.GMS: Cournot and Bertrand with continuous strategies

$ONTEXT
begin with Cournot doupoly
single unified market, constant marginal costs
goods XH and XF are imperfect substitutes
inverse demand functions PH = ALPHA - BETA*XH - GAMMA*XF   BETA > GAMMA
maximizing profits gives FOC (implicity reaction functions)
PROFIT = PH*XH - CH*XH = (ALPHA - BETA*XH - GAMMA*XF)*XH - CH*XH
first order condition:  ALPHA - 2*BETA*XH - GAMMA*XF - CH = 0
$OFFTEXT

PARAMETERS
 ALPHA   intercept of demand curve
 BETA    slope of inverse demand curve wrt own quantity
 GAMMA   slope of inverse demand curve wrt rival's quantity
 CH      marginal cost of home firm
 CF      marginal cost of foreign firm
 WELHC0  welfare in country h before policy under Cournot
 WELHB0  welfare in country h before policy under Bertrand;

ALPHA = 12;
BETA = 2;
GAMMA = 1;
CH = 2;
CF = 2;
```

```
NONNEGATIVE VARIABLES
 PH          price of XH
 PF          price of XF
 XH          quantity of XH
 XF          quantity of XF
 PROFH       profit of firm h
 PROFF       profit of firm f;

EQUATIONS
 PRICEH      inverse demand curve facing firm h
 PRICEF      inverse demand curve facing firm f
 HCOURNOT    cournot FOC for firm h (reaction function)
 FCOURNOT    cournot FOC for firm f (reaction function)
 PROFITH     profit of firm h
 PROFITF     profit of firm f;

PRICEH..     PH =E= ALPHA - BETA*XH - GAMMA*XF;

PRICEF..     PF =E= ALPHA - BETA*XF - GAMMA*XH;

HCOURNOT..   CH =G= ALPHA - 2*BETA*XH - GAMMA*XF;

FCOURNOT..   CF =G= ALPHA - 2*BETA*XF - GAMMA*XH;

PROFITH..    PROFH =E= PH*XH - CH*XH;
```

```
PROFITF..   PROFF =E= PF*XF - CF*XF;

MODEL COURNOT  /PRICEH.PH, PRICEF.PF, HCOURNOT.XH, FCOURNOT.XF,
                 PROFITH.PROFH, PROFITF.PROFF/;

SOLVE COURNOT USING MCP;

WELHC0 = PROFH.L;


$ONTEXT
now assume Bertrand price competition
requires you to invert the inverse demand functions
XH = INTERB - SLOPEB1*PH + SLOPEB2*PF
$OFFTEXT

PARAMETERS

 INTERB    intercept of the (direct) demand function
 SLOPEB1   slope of the demand function wrt own price
 SLOPEB2   slope of the demand function wrt rival's price;

INTERB =  (ALPHA*BETA - ALPHA*GAMMA)/(BETA**2 - GAMMA**2);
SLOPEB1 = BETA/(BETA**2 - GAMMA**2);
SLOPEB2 = GAMMA/(BETA**2 - GAMMA**2);
```

```
EQUATIONS
 XBERTH     demand for XH
 XBERTF     demand for XF
 HBERTRAND  bertrand FOC for PH
 FBERTRAND  bertrand FOC for PF;

XBERTH..    XH =E= INTERB - SLOPEB1*PH + SLOPEB2*PF;

XBERTF..    XF =E= INTERB - SLOPEB1*PF + SLOPEB2*PH;

HBERTRAND.. -SLOPEB1*CH =E= INTERB - 2*SLOPEB1*PH + SLOPEB2*PF;

FBERTRAND.. -SLOPEB1*CF =E= INTERB - 2*SLOPEB1*PF + SLOPEB2*PH;

MODEL BERTRAND  /XBERTH.XH, XBERTF.XF, HBERTRAND.PH, FBERTRAND.PF,
                 PROFITH.PROFH, PROFITF.PROFF/;

SOLVE BERTRAND USING MCP;

WELHB0 = PROFH.L;


*now analyze a production subsidy by h (strategic trade policy)
```

```
PARAMETER
 S  subsidy on H's output
 WELFAREHC  country h's welfare under Cournot
 WELFAREHB  country h's welfare under Bertrand;

S = 0.4;
CH = CH - S;

SOLVE COURNOT USING MCP;
WELFAREHC = PROFH.L - S*XH.L;
DISPLAY WELHC0, WELFAREHC;


SOLVE BERTRAND USING MCP;
WELFAREHB = PROFH.L - S*XH.L;
DISPLAY WELHC0, WELFAREHC,WELHB0, WELFAREHB;


$ONTEXT
now let's use nlp to find the OPTIMAL subsidies under Cournot and
Bertrand keep in mind that the optimal subsidy may be NEGATIVE, meaning
a tax let's play the goofy Brander-Spencer gams that all output is sold
to a third country.  Then welfare = profits minus subsidy payments
or plus tax payments.  PROFF will give the welfare of country f
$OFFTEXT
```

```
CH = 2;
S = 0;

VARIABLES
 WELFJ    joint welfare
 SUBH     subsidy on XH is now a (free) variable: it can be negative
 WELHS    welfare of country h: objective to maximize
 SUBF     subsidy on XF is now a (free) variable: it can be negative
 WELFS    welfare of country F: objective to maximize;

EQUATIONS
 WELJ          joint welfare - Cobb-Douglas
 WELH          welfare of country h is WELH = PROFH - SUBH*XH
 PROFITHS      new equation for profits of firm h - replaces PROFITH
 WELF          welfare of country f is WELF = PROFF - SUBF*XF
 PROFITFS      new equation for profits of firm f - replaces PROFITF
 HCOURNOTS     new Cournot reaction function firm h - replaces HCOURNOT
 HBERTRANDS    new Bertrand reaction function firm h - replaces HBERTRAND
 FCOURNOTS     new Cournot reaction function firm f - replaces FCOURNOT
 fBERTRANDS    new Bertrand reaction function firm f - replaces fBERTRAND;

WELJ..         WELFJ =E= WELHS**0.5*WELFS**0.5;

WELH..         WELHS =E= PROFH - SUBH*XH;

PROFITHS..     PROFH =E= PH*XH - (CH - SUBH)*XH;
```

```
HCOURNOTS..  (CH - SUBH) =E= ALPHA - 2*BETA*XH - GAMMA*XF;

HBERTRANDS.. -SLOPEB1*(CH-SUBH) =E= INTERB - 2*SLOPEB1*PH + SLOPEB2*PF;

WELF..       WELFS =E= PROFF - SUBF*XF;

PROFITFS..   PROFF =E= PF*XF - (CF - SUBF)*XF;

FCOURNOTS..  (CF - SUBF) =E= ALPHA - 2*BETA*XF - GAMMA*XH;

FBERTRANDS.. -SLOPEB1*(CF-SUBF) =E= INTERB - 2*SLOPEB1*PF + SLOPEB2*PH;

SUBH.L = 0.4;
WELHS.L = 8;



* first, a unilateral action by the government of country h

SUBF.FX = 0;

MODEL COURNOTS  /WELH, HCOURNOTS, FCOURNOT, PRICEH, PRICEF,
                 PROFITHS, PROFITF/;
SOLVE COURNOTS  USING NLP MAXIMIZING WELHS;
```

```
MODEL BERTRANDS /WELH, HBERTRANDS, FBERTRAND, XBERTH, XBERTF,
                 PROFITHS, PROFITF/;
SOLVE BERTRANDS USING NLP MAXIMIZING WELHS;

SUBF.UP = +INF;
SUBF.LO = -INF;

* compute cooperative and non-cooperative outcomes between governments

SETS I /I1*I10/
     J /COOP, NONCOOP/;

PARAMETER
 RESULTSC(*, J);


* compute a cooperative Nash eq between the governments

MODEL WELFJOINT /WELJ, WELH, WELF, HCOURNOTS, FCOURNOTS, PRICEH, PRICEF,
                 PROFITHS, PROFITFS/;
SOLVE WELFJOINT USING NLP MAXIMIZING WELFJ;


RESULTSC("WELJ", "COOP") = WELFJ.L;
RESULTSC("WELH", "COOP") = WELHS.L;
RESULTSC("WELF", "COOP") = WELFS.L;
```

```
RESULTSC("PROFITH", "COOP") = PROFH.L;
RESULTSC("PROFITF", "COOP") = PROFF.L;
RESULTSC("SUBH", "COOP") = SUBH.L;
RESULTSC("SUBF", "COOP") = SUBF.L;

DISPLAY RESULTSC;


* compute a non-cooperative outcome in subsidy rates
* iterative procedure:
*  max WELHS subject to SUBF fixed
*  hold SUBH at it's solution level and free up SUBF
*  max WELFS solve model for fixed SUBH
*  repeat 10 time

SUBH.L = 0;
SUBF.L = 0;

LOOP(I,

SUBH.LO = -INF;
SUBH.UP = +INF;
SUBF.FX = SUBF.L;

SOLVE WELFJOINT USING NLP MAXIMIZING WELHS;
```

```
SUBF.LO = -INF;
SUBF.UP = +INF;
SUBH.FX = SUBH.L;

SOLVE WELFJOINT USING NLP MAXIMIZING WELFS;

);

RESULTSC("WELJ", "NONCOOP") = WELFJ.L;
RESULTSC("WELH", "NONCOOP") = WELHS.L;
RESULTSC("WELF", "NONCOOP") = WELFS.L;
RESULTSC("PROFITH", "NONCOOP") = PROFH.L;
RESULTSC("PROFITF", "NONCOOP") = PROFF.L;
RESULTSC("SUBH", "NONCOOP") = SUBH.L;
RESULTSC("SUBF", "NONCOOP") = SUBF.L;

DISPLAY RESULTSC;
```

```
$TITLE: M4-1.GMS: Cournot and Bertrand with continuous strategies

$ONTEXT
begin with Cournot doupoly
single unified market, constant marginal costs
goods XH and XF are imperfect substitutes
inverse demand functions PH = ALPHA - BETA*XH - GAMMA*XF   BETA > GAMMA
maximizing profits gives FOC (implicity reaction functions)
PROFIT = PH*XH - CH*XH = (ALPHA - BETA*XH - GAMMA*XF)*XH - CH*XH
first order condition:  ALPHA - 2*BETA*XH - GAMMA*XF - CH = 0
$OFFTEXT

PARAMETERS
 ALPHA   intercept of demand curve
 BETA    slope of inverse demand curve wrt own quantity
 GAMMA   slope of inverse demand curve wrt rival's quantity
 CH      marginal cost of home firm
 CF      marginal cost of foreign firm
 WELHC0  welfare in country h before policy under Cournot
 WELHB0  welfare in country h before policy under Bertrand;

ALPHA = 12;
BETA = 2;
GAMMA = 1;
CH = 2;
CF = 2;
```

```
NONNEGATIVE VARIABLES
 PH          price of XH
 PF          price of XF
 XH          quantity of XH
 XF          quantity of XF
 PROFH       profit of firm h
 PROFF       profit of firm f;

EQUATIONS
 PRICEH      inverse demand curve facing firm h
 PRICEF      inverse demand curve facing firm f
 HCOURNOT    cournot FOC for firm h (reaction function)
 FCOURNOT    cournot FOC for firm f (reaction function)
 PROFITH     profit of firm h
 PROFITF     profit of firm f;

PRICEH..    PH =E= ALPHA - BETA*XH - GAMMA*XF;

PRICEF..    PF =E= ALPHA - BETA*XF - GAMMA*XH;

HCOURNOT..  CH =G= ALPHA - 2*BETA*XH - GAMMA*XF;

FCOURNOT..  CF =G= ALPHA - 2*BETA*XF - GAMMA*XH;

PROFITH..   PROFH =E= PH*XH - CH*XH;
```

```
PROFITF..   PROFF =E= PF*XF - CF*XF;

MODEL COURNOT  /PRICEH.PH, PRICEF.PF, HCOURNOT.XH, FCOURNOT.XF,
                 PROFITH.PROFH, PROFITF.PROFF/;

SOLVE COURNOT USING MCP;

WELHC0 = PROFH.L;


$ONTEXT
now assume Bertrand price competition
requires you to invert the inverse demand functions
XH = INTERB - SLOPEB1*PH + SLOPEB2*PF
$OFFTEXT

PARAMETERS

 INTERB    intercept of the (direct) demand function
 SLOPEB1   slope of the demand function wrt own price
 SLOPEB2   slope of the demand function wrt rival's price;

INTERB =  (ALPHA*BETA - ALPHA*GAMMA)/(BETA**2 - GAMMA**2);
SLOPEB1 = BETA/(BETA**2 - GAMMA**2);
SLOPEB2 = GAMMA/(BETA**2 - GAMMA**2);
```

```
EQUATIONS
 XBERTH     demand for XH
 XBERTF     demand for XF
 HBERTRAND  bertrand FOC for PH
 FBERTRAND  bertrand FOC for PF;

XBERTH..    XH =E= INTERB - SLOPEB1*PH + SLOPEB2*PF;

XBERTF..    XF =E= INTERB - SLOPEB1*PF + SLOPEB2*PH;

HBERTRAND.. -SLOPEB1*CH =E= INTERB - 2*SLOPEB1*PH + SLOPEB2*PF;

FBERTRAND.. -SLOPEB1*CF =E= INTERB - 2*SLOPEB1*PF + SLOPEB2*PH;

MODEL BERTRAND  /XBERTH.XH, XBERTF.XF, HBERTRAND.PH, FBERTRAND.PF,
                 PROFITH.PROFH, PROFITF.PROFF/;

SOLVE BERTRAND USING MCP;

WELHB0 = PROFH.L;


*now analyze a production subsidy by h (strategic trade policy)
```

```
PARAMETER
 S  subsidy on H's output
 WELFAREHC  country h's welfare under Cournot
 WELFAREHB  country h's welfare under Bertrand;

S = 0.4;
CH = CH - S;

SOLVE COURNOT USING MCP;
WELFAREHC = PROFH.L - S*XH.L;
DISPLAY WELHC0, WELFAREHC;


SOLVE BERTRAND USING MCP;
WELFAREHB = PROFH.L - S*XH.L;
DISPLAY WELHC0, WELFAREHC,WELHB0, WELFAREHB;


$ONTEXT
now let's use nlp to find the OPTIMAL subsidies under Cournot and
Bertrand keep in mind that the optimal subsidy may be NEGATIVE, meaning
a tax let's play the goofy Brander-Spencer gams that all output is sold
to a third country.  Then welfare = profits minus subsidy payments
or plus tax payments.  PROFF will give the welfare of country f
$OFFTEXT
```

```
CH = 2;
S = 0;

VARIABLES
 WELFJ    joint welfare
 SUBH     subsidy on XH is now a (free) variable: it can be negative
 WELHS    welfare of country h: objective to maximize
 SUBF     subsidy on XF is now a (free) variable: it can be negative
 WELFS    welfare of country F: objective to maximize;

EQUATIONS
 WELJ         joint welfare - Cobb-Douglas
 WELH         welfare of country h is WELH = PROFH - SUBH*XH
 PROFITHS     new equation for profits of firm h - replaces PROFITH
 WELF         welfare of country f is WELF = PROFF - SUBF*XF
 PROFITFS     new equation for profits of firm f - replaces PROFITF
 HCOURNOTS    new Cournot reaction function firm h - replaces HCOURNOT
 HBERTRANDS   new Bertrand reaction function firm h - replaces HBERTRAND
 FCOURNOTS    new Cournot reaction function firm f - replaces FCOURNOT
 fBERTRANDS   new Bertrand reaction function firm f - replaces fBERTRAND;

WELJ..        WELFJ =E= WELHS**0.5*WELFS**0.5;

WELH..        WELHS =E= PROFH - SUBH*XH;

PROFITHS..    PROFH =E= PH*XH - (CH - SUBH)*XH;
```

```
HCOURNOTS..  (CH - SUBH) =E= ALPHA - 2*BETA*XH - GAMMA*XF;

HBERTRANDS.. -SLOPEB1*(CH-SUBH) =E= INTERB - 2*SLOPEB1*PH + SLOPEB2*PF;

WELF..       WELFS =E= PROFF - SUBF*XF;

PROFITFS..   PROFF =E= PF*XF - (CF - SUBF)*XF;

FCOURNOTS..  (CF - SUBF) =E= ALPHA - 2*BETA*XF - GAMMA*XH;

FBERTRANDS.. -SLOPEB1*(CF-SUBF) =E= INTERB - 2*SLOPEB1*PF + SLOPEB2*PH;

SUBH.L = 0.4;
WELHS.L = 8;



* first, a unilateral action by the government of country h

SUBF.FX = 0;

MODEL COURNOTS  /WELH, HCOURNOTS, FCOURNOT, PRICEH, PRICEF,
                 PROFITHS, PROFITF/;
SOLVE COURNOTS  USING NLP MAXIMIZING WELHS;
```

```
MODEL BERTRANDS /WELH, HBERTRANDS, FBERTRAND, XBERTH, XBERTF,
                 PROFITHS, PROFITF/;
SOLVE BERTRANDS USING NLP MAXIMIZING WELHS;

SUBF.UP = +INF;
SUBF.LO = -INF;

* compute cooperative and non-cooperative outcomes between governments

SETS I /I1*I10/
     J /COOP, NONCOOP/;

PARAMETER
 RESULTSC(*, J);


* compute a cooperative Nash eq between the governments

MODEL WELFJOINT /WELJ, WELH, WELF, HCOURNOTS, FCOURNOTS, PRICEH, PRICEF,
                 PROFITHS, PROFITFS/;
SOLVE WELFJOINT USING NLP MAXIMIZING WELFJ;


RESULTSC("WELJ", "COOP") = WELFJ.L;
RESULTSC("WELH", "COOP") = WELHS.L;
RESULTSC("WELF", "COOP") = WELFS.L;
```

```
RESULTSC("PROFITH", "COOP") = PROFH.L;
RESULTSC("PROFITF", "COOP") = PROFF.L;
RESULTSC("SUBH", "COOP") = SUBH.L;
RESULTSC("SUBF", "COOP") = SUBF.L;

DISPLAY RESULTSC;


* compute a non-cooperative outcome in subsidy rates
* iterative procedure:
*  max WELHS subject to SUBF fixed
*  hold SUBH at it's solution level and free up SUBF
*  max WELFS solve model for fixed SUBH
*  repeat 10 time

SUBH.L = 0;
SUBF.L = 0;

LOOP(I,

SUBH.LO = -INF;
SUBH.UP = +INF;
SUBF.FX = SUBF.L;

SOLVE WELFJOINT USING NLP MAXIMIZING WELHS;
```

```
SUBF.LO = -INF;
SUBF.UP = +INF;
SUBH.FX = SUBH.L;

SOLVE WELFJOINT USING NLP MAXIMIZING WELFS;

);

RESULTSC("WELJ", "NONCOOP") = WELFJ.L;
RESULTSC("WELH", "NONCOOP") = WELHS.L;
RESULTSC("WELF", "NONCOOP") = WELFS.L;
RESULTSC("PROFITH", "NONCOOP") = PROFH.L;
RESULTSC("PROFITF", "NONCOOP") = PROFF.L;
RESULTSC("SUBH", "NONCOOP") = SUBH.L;
RESULTSC("SUBF", "NONCOOP") = SUBF.L;

DISPLAY RESULTSC;
```

4.2 Nash equilibria with discrete strategies

Gams has some great set features that allow a modeler to lots of very interesting economics.

Here, I present a simple example of a two-player normal-form game in which each player has three strategies.

This particular version is motivated by a two-country trade model with multinational firms in which there is one firm in each country. Each firm may:

- not enter, strategy 0
- enter with a single plant at home, exporting to the other country, strategy 1
- enter with plants in both countries, serving each market from a local plant, strategy 2

In an actual model, the numerical values in the payoff matrices are solved for from the underlying duopoly problem. Here I'll just make up number consistent with the underlying example.

```
SETS R  strategies for firm h /SH0, SH1, SH2/
     C  strategies for firm f /SF0, SF1, SF2/;

ALIAS(R,RR)
ALIAS(C,CC);

TABLE PAYOFFH(*,*)
      SF0   SF1   SF2
SH0   -.1   -.1   -.1
SH1    10     6     3
SH2    12     5     2 ;

TABLE PAYOFFF(*,*)
      SF0   SF1   SF2
SH0   -.1    10    12
SH1   -.1     6     5
SH2   -.1     3     2 ;
```

A best response Nash equilibrium, involves player h picking the row element that is the largest given the column selected by player f and vice versa (f picks the highest column for h's row pick).

There is GAMS command that identifies the best response strategy. First, some GAMS notation.

X = 1$(Y EQ 1) means:

"set X equal to one if Y is equal to one, otherwise set X = 0"

X$(Y EQ 1) = 1 means something subtlely different:

"set X equal to one if Y is equal to one, otherwise leave the existing value of X unchanged"

We will need the first version here.

Let ROWMAX(R,C) be a matrix for h.

A value = 1 in cell (R,C) will denote h's best response row R when f plays column strategy C. Non-optimal responses = 0.

Let COLMAX(R,C) be a matrix for f.

A value = 1 in cell (R,C) will denote f's best response column C when h plays row strategy R. Non-optimal responses = 0.

The crucial GAMS command is SMAX (set max):

SMAX(RR, PAYOFF(RR,C)) is the maximum value of the parameter PAYOFF over the rows, for a given column C

The best-response matrices with zeros and ones are given by:

```
ROWMAX(R,C) = 1$(PAYOFFH(R,C) EQ SMAX(RR, PAYOFFH(RR,C)));

COLMAX(R,C) = 1$(PAYOFFF(R,C) EQ SMAX(CC, PAYOFFF(R,CC)));
```

Now multiple these two matrices together element by element, to get a new matrix NASHEQ(R,C).

A one denote a best response for both h and f, and hence that (R,C) cell is a Nash equilibrium.

```
NASHEQ(R,C) = ROWMAX(R,C)*COLMAX(R,C);
```

Finally, the profits at each Nash equilibrium are given by

```
PROFHNE(R,C) = PAYOFFH(R,C)$NASHEQ(R,C);

PROFFNE(R,C) = PAYOFFF(R,C)$NASHEQ(R,C);
```

This technique will find ALL pure-strategy Nash equilibria. The second example shows a case of multiple equilibria.

Case 1: each firm chooses one plant and exports to the other country (1,1), exporting duopoly shown above

Case 2: three equilibria. Exporting duopoly as in Case 1, or one firm chooses two plants, and the other firm does not enter: (1,1), (2,0), (0,2)

Case 3: each firm chooses two plants, a horizontal multinational duopoly (2,2)

```
$TITLE: M4-2.GMS: Find all pure-strategy Nash equilibrium
*                 with discrete strategy sets

$ONTEXT
two firms, one in country h and one in country f
each firm chooses one of three strategies:
 don't enter: strategy 0
 enter with a single plant and export to the other country: strategy 1
 enter with plants in both countries (horizontal multinational)
   strategy 2
$OFFTEXT

SETS R  strategies for firm h /SH0, SH1, SH2/
     C  strategies for firm f /SF0, SF1, SF2/;

ALIAS(R,RR)
ALIAS(C,CC);

PARAMETERS
 ROWMAX(R,C)    maximum value over the rows for a given column C
 COLMAX(R,C)    maximum value over the columns for a given row R
 NASHEQ(R,C)    matrix of 0-1 where 1 is a Nash equilibrium
 PROFHNE(R,C)   profit of firm h in Nash equilibrium
 PROFFNE(R,C)   profit of firm f in Nash equilibrium;

* small maintenance costs -0.1 when not entering, not needed
```

```
TABLE PAYOFFH(*,*)
      SF0   SF1   SF2
SH0   -.1   -.1   -.1
SH1    10     6     3
SH2    12     5     2 ;

TABLE PAYOFFF(*,*)
      SF0   SF1   SF2
SH0   -.1    10    12
SH1   -.1     6     5
SH2   -.1     3     2 ;


DISPLAY PAYOFFH, PAYOFFF;

ROWMAX(R,C) = 1$(PAYOFFH(R,C) EQ SMAX(RR, PAYOFFH(RR,C)));
COLMAX(R,C) = 1$(PAYOFFF(R,C) EQ SMAX(CC, PAYOFFF(R,CC)));

DISPLAY ROWMAX, COLMAX;

NASHEQ(R,C) = ROWMAX(R,C)*COLMAX(R,C);

DISPLAY NASHEQ;

PROFHNE(R,C) = PAYOFFH(R,C)$NASHEQ(R,C);
```

```
PROFFNE(R,C) = PAYOFFF(R,C)$NASHEQ(R,C);

DISPLAY PROFHNE, PROFFNE;


*CASE 2: MARKETS TOO SMALL FOR A FIRM TO ENTER AGAINST A TWO-PLANT RIVAL
*subtract 4 from each payoff strategies 1 and 2

TABLE PAYOFFH2(*,*)
      SF0   SF1   SF2
SH0   -.1   -.1   -.1
SH1     6     2    -1
SH2     8     1    -2 ;

TABLE PAYOFFF2(*,*)
      SF0   SF1   SF2
SH0   -.1     6     8
SH1   -.1     2     1
SH2   -.1    -1    -2 ;

ROWMAX(R,C) = 1$(PAYOFFH2(R,C) EQ SMAX(RR, PAYOFFH2(RR,C)));
COLMAX(R,C) = 1$(PAYOFFF2(R,C) EQ SMAX(CC, PAYOFFF2(R,CC)));

NASHEQ(R,C) = ROWMAX(R,C)*COLMAX(R,C);

DISPLAY NASHEQ;
```

```
PROFHNE(R,C) = PAYOFFH2(R,C)$NASHEQ(R,C);
PROFFNE(R,C) = PAYOFFF2(R,C)$NASHEQ(R,C);

DISPLAY PROFHNE, PROFFNE;


*CASE 3: LOWER FIRM FIXED COSTS, RAISE PLANT FIXED COSTS
*makes two-plant production more profitable
*add 2 when playing strategy 2

TABLE PAYOFFH3(*,*)
      SF0   SF1   SF2
SH0   -.1   -.1   -.1
SH1    10     6     3
SH2    14     7     4 ;

TABLE PAYOFFF3(*,*)
      SF0   SF1   SF2
SH0   -.1    10    14
SH1   -.1     6     7
SH2   -.1     3     4 ;

ROWMAX(R,C) = 1$(PAYOFFH3(R,C) EQ SMAX(RR, PAYOFFH3(RR,C)));
COLMAX(R,C) = 1$(PAYOFFF3(R,C) EQ SMAX(CC, PAYOFFF3(R,CC)));
```

```
NASHEQ(R,C) = ROWMAX(R,C)*COLMAX(R,C);

DISPLAY NASHEQ;

PROFHNE(R,C) = PAYOFFH3(R,C)$NASHEQ(R,C);
PROFFNE(R,C) = PAYOFFF3(R,C)$NASHEQ(R,C);

DISPLAY PROFHNE, PROFFNE;
```

4.3 An insurance problem illustrating moral hazzard and adverse selection

Consumers can buy insurance against the risk of future sickness or an accident.

Consumers can reduce their risk of sickness/accident through EFFORT, though effort has a utility cost.

Consumers have different inherently riskiness parameters (teenagers versus old folks), referred to as their TYPE.

Insurance company cannot observe a customer’s type nor their effort. Insurance company only knows the distribution of types.

Moral Hazzard: buying insurance makes the consumer less careful (exerts less effort).

Adverse selection: more risky individuals want to buy more insurance, safe types may not buy at all.

A consumer has (exogenous parameters for income (M) and risk type (TYPE). Consumers face a fixed price for in insurance and can buy as little or as much as they like.

```
                                                   |---Sick
        |--Do not buy Insurance--choose effort --|
        |                                         |---Healthy
Income--|
+ Type  |                                         |---Sick
        |--   Buy Insurance    --choose effort---|
                                                  |---Healthy
```

The insurance company knows only the distribution of riskiness of the population. Must offer insurance at a single price to all customers.

We use two consumer types: TYPE = probability of not being sick at zero effort. RISKAV = ((1-TYPE1) + (1-TYPE2))/2

```
TYPE      risk type: probability of good health at effort = 0
 RISKAV   average riskiness at effort = 0
 M0       income in the first time period
 MH       income in the second time period when healthy
 MS       income in the second time period when sick (before insur)
 ACUF     actuarially fairness: 1 = actuarily fair, ACUF < 1 unfair
 BETA     makes utility of consumption concave;

* BETA also interpreted as constant relative risk aversion.

NONNEGATIVE VARIABLES
 INS      insurance purchased
 PNS      payoff from insurance when sick
 ALPHA    probability of good health
 EFFORT   effort spent to insure good health: diet, exercise;
```

```
VARIABLES

 U        expected utility;

EQUATIONS

 UTILITY   the utility of having or not having insurance
 INSURANCE the amount of insurance puchased (INS), payoff (PNS)
 MORALHAZ  relationship between effort and prob of being healthy;


UTILITY..     U =E= (M0-INS)**BETA
                    + ALPHA*MH**BETA + (1-ALPHA)*(MS+PNS)**BETA
                    - (0.06)*(EFFORT + EFFORT**2);

INSURANCE..   INS*ACUF =E= PNS*RISKAV;

MORALHAZ..    ALPHA =E= TYPE + 0.15*EFFORT;


MODEL OPTIMIZE /UTILITY, INSURANCE, MORALHAZ/;
```

Several experiments: (a) one risk type = 0.5, (b) two risk types =0.55 and 0.45.

```
$TITLE: M4-3a.GMS: modeling health insurance
*    with moral hazzard, adverse selection modeled as a NLP
* COPYRIGHT JAMES R. MARKUSEN AND DAVID WHITEHILL

$ONTEXT
MODELING DEMAND FOR HEALTH INSURANCE
                                                       |---Sick
         |--Do not buy Insurance--choose effort --|
         |                                        |---Healthy
Income---|
+ Type   |                                        |---Sick
         |--   Buy Insurance    --choose effort---|
                                                  |---Healthy
$OFFTEXT

PARAMETERS
 TYPE     risk type: probability of good health at effort = 0
 RISKAV   average riskiness at effort = 0
 M0       income in the first time period
 MH       income in the second time period when healthy
 MS       income in the second time period when sick (before insurance)
 ACUF     acutuarially fairness   1 = actuarily fair  ACUF < 1 unfair
 BETA     needed to make the consumption concave(diminishing returns)
 INS0, PNS0, ALPHA0, EFFORT0, PROFIT0  store results for single type
 INS1, PNS1, ALPHA1, EFFORT1, PROFIT1  store results for type 1 (safe)
 INS2, PNS2, ALPHA2, EFFORT2, PROFIT2  store results for type 2 (risky)
```

```
 PROFIT    profit of the insurance firm selling to both types;

ACUF=1.0;
BETA = 0.5;
M0 = 10;
MH = 10;
MS = 4;
TYPE = 0.5;
RISKAV = 1-TYPE;

NONNEGATIVE VARIABLES
 INS       insurance purchased
 PNS       payoff from insurance when sick
 ALPHA     probability of good health
 EFFORT    effort spent to insure good health: diet exercise and such;

VARIABLES

 U         expected utility;

EQUATIONS

 UTILITY   the utility of having or not having insurance
 INSURANCE the amount of insurance puchased (INS) and the payoff (PNS)
 MORALHAZ  relationship between effort and probability of being healthy;
```

```
* the coefficient 0.06 is chosen so that effort is marginally non-optima»
l
* in the benchmark with actuarily fair insurance

UTILITY..    U =E= (M0-INS)**BETA
                   + ALPHA*MH**BETA + (1-ALPHA)*(MS+PNS)**BETA
                   - (0.06)*(EFFORT + EFFORT**2);

INSURANCE..  INS*ACUF =E= PNS*RISKAV;

MORALHAZ..   ALPHA =E= TYPE + 0.15*EFFORT;


MODEL OPTIMIZE /UTILITY, INSURANCE, MORALHAZ/;

U.L = 1;
INS.L =2;
PNS.L = 1;
ALPHA.L = 0.5;
EFFORT.L = 0;

*solve first for single type
TYPE = 0.5;

SOLVE OPTIMIZE USING NLP MAXIMIZING U;
```

```
INS0 = INS.L;
PNS0 = PNS.L;
ALPHA0 = ALPHA.L;
EFFORT0 = EFFORT.L;

PROFIT0 = INS0 - (1 - ALPHA0)*PNS0;

DISPLAY INS0, PNS0, ALPHA0, EFFORT0, PROFIT0;

*now assume two types, solve first for the safe type
TYPE = 0.55;
RISKAV = ((1-0.55)+(1-0.45))/2;

SOLVE OPTIMIZE USING NLP MAXIMIZING U;

INS1 = INS.L;
PNS1 = PNS.L;
ALPHA1 = ALPHA.L;
EFFORT1 = EFFORT.L;

PROFIT1 = INS1 - (1 - ALPHA1)*PNS1;

DISPLAY INS1, PNS1, ALPHA1, EFFORT1, PROFIT1;

*solve for the risky type
TYPE = 0.45;
```

```
SOLVE OPTIMIZE USING NLP MAXIMIZING U;

INS2 = INS.L;
PNS2 = PNS.L;
ALPHA2 = ALPHA.L;
EFFORT2 = EFFORT.L;

PROFIT2 = INS2 - (1 - ALPHA2)*PNS2;

DISPLAY INS0, PNS0, ALPHA0, EFFORT0, PROFIT0;
DISPLAY INS1, PNS1, ALPHA1, EFFORT1, PROFIT1;
DISPLAY INS2, PNS2, ALPHA2, EFFORT2, PROFIT2;

PROFIT = PROFIT1 + PROFIT2;
DISPLAY PROFIT;

*$exit

* generate some scenarios

SETS I /I1*I8/;

PARAMETERS
 RESULTS(I, *);
```

```
TYPE = 0.5;
RISKAV = 1-TYPE;

SOLVE OPTIMIZE USING NLP MAXIMIZING U;
RESULTS("I1", "INS")  = INS.L;
RESULTS("I1", "ALPHA")  = ALPHA.L;
RESULTS("I1", "EFFORT") = EFFORT.L;
RESULTS("I1", "ACUF") = ACUF;
RESULTS("I1", "IS")  = MS;
RESULTS("I1", "BETA")  = BETA;

*Actuarially unfair added
ACUF = 0.8;

SOLVE OPTIMIZE USING NLP MAXIMIZING U;
RESULTS("I2", "INS")  = INS.L;
RESULTS("I2", "ALPHA")  = ALPHA.L;
RESULTS("I2", "EFFORT") = EFFORT.L;
RESULTS("I2", "ACUF") = ACUF;
RESULTS("I2", "IS")  = MS;
RESULTS("I2", "BETA")  = BETA;

*Loss from getting sick is higher

ACUF = 1.0;
MS = 2;
```

```
SOLVE OPTIMIZE USING NLP MAXIMIZING U;
RESULTS("I3", "INS")  = INS.L;
RESULTS("I3", "ALPHA")  = ALPHA.L;
RESULTS("I3", "EFFORT") = EFFORT.L;
RESULTS("I3", "ACUF") = ACUF;
RESULTS("I3", "IS")  = MS;
RESULTS("I3", "BETA")  = BETA;

ACUF = 0.8;
MS = 2;

SOLVE OPTIMIZE USING NLP MAXIMIZING U;
RESULTS("I4", "INS")  = INS.L;
RESULTS("I4", "ALPHA")  = ALPHA.L;
RESULTS("I4", "EFFORT") = EFFORT.L;
RESULTS("I4", "ACUF") = ACUF;
RESULTS("I4", "IS")  = MS;
RESULTS("I4", "BETA")  = BETA;

*Risk aversion is higher, actuarily fair

MS = 4;
BETA = 0.4;
ACUF = 1.0;
INS.L = 2.5;
```

```
SOLVE OPTIMIZE USING NLP MAXIMIZING U;
RESULTS("I5", "INS")  = INS.L;
RESULTS("I5", "ALPHA")  = ALPHA.L;
RESULTS("I5", "EFFORT") = EFFORT.L;
RESULTS("I5", "ACUF") = ACUF;
RESULTS("I5", "IS")  = MS;
RESULTS("I5", "BETA")  = BETA;

*Risk aversion higher, actuarily unfair

MS = 4;
BETA = 0.4;
ACUF = 0.8;

SOLVE OPTIMIZE USING NLP MAXIMIZING U;
RESULTS("I6", "INS")  = INS.L;
RESULTS("I6", "ALPHA")  = ALPHA.L;
RESULTS("I6", "EFFORT") = EFFORT.L;
RESULTS("I6", "ACUF") = ACUF;
RESULTS("I6", "IS")  = MS;
RESULTS("I6", "BETA")  = BETA;

*Risk aversion is higher, actuarily fair, lower MS

MS = 2;
```

```
BETA = 0.4;
ACUF = 1.0;

SOLVE OPTIMIZE USING NLP MAXIMIZING U;
RESULTS("I7", "INS")  = INS.L;
RESULTS("I7", "ALPHA")  = ALPHA.L;
RESULTS("I7", "EFFORT") = EFFORT.L;
RESULTS("I7", "ACUF") = ACUF;
RESULTS("I7", "IS")  = MS;
RESULTS("I7", "BETA")  = BETA;

*Risk aversion higher, actuarily unfair, lower MS

MS = 2;
BETA = 0.4;
ACUF = 0.8;

SOLVE OPTIMIZE USING NLP MAXIMIZING U;
RESULTS("I8", "INS")  = INS.L;
RESULTS("I8", "ALPHA")  = ALPHA.L;
RESULTS("I8", "EFFORT") = EFFORT.L;
RESULTS("I8", "ACUF") = ACUF;
RESULTS("I8", "IS")  = MS;
RESULTS("I8", "BETA")  = BETA;

DISPLAY RESULTS;
```

```
results for single type = 0.5

----     114 PARAMETER INS0                  =        2.000
             PARAMETER PNS0                  =        4.000
             PARAMETER ALPHA0                =        0.500
             PARAMETER EFFORT0               =        0.000
             PARAMETER PROFIT0               =        0.000

results for type 1 safe: type = 0.55

----     115 PARAMETER INS1                  =        0.000
             PARAMETER PNS1                  =        0.000
             PARAMETER ALPHA1                =        0.693
             PARAMETER EFFORT1               =        0.953
             PARAMETER PROFIT1               =        0.000

results for type 2 risky: type = 0.45

----     116 PARAMETER INS2                  =        2.523
             PARAMETER PNS2                  =        5.047
             PARAMETER ALPHA2                =        0.450
             PARAMETER EFFORT2               =        0.000
             PARAMETER PROFIT2               =       -0.252

profit from selling to both types at actuarial average 0.5

----     119 PARAMETER PROFIT                =       -0.252
```

here are results for a single consumer type with TYPE = 0.5, considering different values of three parameters

exogenous parameters

ACUF (actuarial fairness of insurance price)
IS (income when sick)
BETA (lower BETA, more risk averse)

endogenous variables

INS (insurance purchased)
ALPHA (probability of being healthy)
EFFORT (effort to be safe and healthy)

	INS	ALPHA	EFFORT	ACUF	IS	BETA
I1	2.000	0.500		1.000	4.000	0.500
I2		0.643	0.953	0.800	4.000	0.500
I3	2.331	0.534	0.226	1.000	2.000	0.500
I4		0.753	1.685	0.800	2.000	0.500
I5	2.000	0.500		1.000	4.000	0.400
I6	1.105	0.518	0.120	0.800	4.000	0.400
I7	2.667	0.500		1.000	2.000	0.400
I8	1.797	0.543	0.284	0.800	2.000	0.400