Slides for Chapter 7: Adding scale economies and imperfect competition to general equilibrium

7.1 An introduction to Dixit-Stiglitz CES preferences

D-S preferences are a special, symmetric case of CES preferences, elasticity of substitution > 1. A more general treatment of CES will be given later.

Y will be a competitive, constant-returns industry while X will consist of an endogenous number of differentiated varieties.

Utility of the representative consumer in each country is Cobb-

Douglas, and the symmetry of varieties within a group of goods allows us to write utility as follows ($0 < \alpha < 1$).

$$U = X_c^{\beta} Y^{1-\beta}, \qquad X_c \equiv \left[\sum_i^N (X_i)^{\alpha} \right]^{1/\alpha}$$

where the number of varieties N is endogenous.

This function permits the use of two-stage budgeting, in which the consumer first allocates total income (M) between Y and X_c.

Let e denote the minimum cost of buying one unit of X_c at price p for the individual varieties (i.e., e is the unit expenditure function for X_c). Y is numeraire. First-stage budgeting yields:

$$Y = (1-\beta)M \qquad X_c = \beta M/e$$

$$e(p^k) = \min(X_i) \sum_i pX_i \quad st \quad X_c = 1$$

Let $M_x = \beta M$ be the expenditure on X in aggregate. Solve for the demand for a given X variety, and for the price index e.

The consumer's sub-problem maximizing the utility from X goods subject to an expenditure constraint (using λ as a Lagrangean multiplier) and first-order conditions are:

$$\max X_c = \left[\sum X_i^{\alpha}\right]^{\frac{1}{\alpha}} + \lambda(M_x - \sum p_i X_i)$$

$$=> \quad \frac{1}{\alpha}\left[\sum X_i^{\alpha}\right]^{\frac{1}{\alpha} - 1} \alpha X_i^{\alpha - 1} - \lambda p_i = 0$$

Let σ denote the elasticity of substitution among varieties. Dividing the first-order condition for variety i by the one for variety j,

$$\left[\frac{X_i}{X_j}\right]^{\alpha - 1} = \frac{p_i}{p_j} \qquad \frac{X_i}{X_j} = \left[\frac{p_i}{p_j}\right]^{\frac{1}{\alpha - 1}} = \left[\frac{p_i}{p_j}\right]^{-\sigma} \qquad since \quad \sigma = \frac{1}{1 - \alpha}$$

$$X_j = \left[\frac{p_i}{p_j}\right]^{\sigma} X_i \qquad p_j X_j = p_j p_j^{-\sigma} p_i^{\sigma} X_i$$

$$\sum p_j X_j = M_x = \left[\sum p_j^{1 - \sigma}\right] p_i^{\sigma} X_i$$

Inverting this last equation, the demand for an individual variety i:

$$X_i = p_i^{-\sigma}\left[\sum p_j^{1-\sigma}\right]^{-1} M_x \qquad \sigma = \frac{1}{1-\alpha}, \qquad \alpha = \frac{\sigma-1}{\sigma}$$

Use X_i to construct X_c and then solve for e, noting the relationship between α and σ.

$$X_i^{\alpha} = X_i^{\frac{\sigma-1}{\sigma}} = p_i^{1-\sigma}\left[\sum p_j^{1-\sigma}\right]^{\frac{1-\sigma}{\sigma}} M_x^{\alpha}$$

$$\sum X_i^{\alpha} = \left[\sum p_i^{1-\sigma}\right]\left[\sum p_j^{1-\sigma}\right]^{\frac{1-\sigma}{\sigma}} M_x^{\alpha} = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{\sigma}} M_x^{\alpha}$$

$$X_c = \left[\sum X_i^{\alpha}\right]^{\frac{1}{\alpha}} = \left[\sum X_i^{\alpha}\right]^{\frac{\sigma}{\sigma-1}} = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{\sigma-1}} M_x$$

$$e = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$ if all prices equal: $$e = N^{\frac{1}{\sigma-1}} p$$

An increase in the *range* of goods lowers the cost of a unit of utility

Having derived e, we can then use equation (13) in (9) to get the demand for an individual variety.

$$X_i \equiv p_i^{-\sigma} e^{\sigma-1} M_x \qquad since \qquad e^{\sigma-1} = \left[\sum p_j^{1-\sigma}\right]^{-1}$$

Now derive the price elasticity of demand for an individual good.

$$X_i = p_i^{-\sigma}\left(\sum p_j^{1-\sigma}\right)^{-1} M_x \qquad s_i \equiv \frac{p_i X_i}{M_x} = p_i^{1-\sigma}\left(\sum p_j^{1-\sigma}\right)^{-1}$$

$$(\ldots) \equiv \left(\sum p_j^{\,1-\sigma}\right)$$

$$\frac{\partial X_i}{\partial p_i} = -\sigma p_i^{\,-\sigma-1}(\ldots)^{-1} M_x - (1-\sigma) p_i^{\,-\sigma}(\ldots)^{-2} p^{\,-\sigma} M_x$$

$$= -\sigma p_i^{\,-\sigma-1}(\ldots)^{-1} M_x + (\sigma-1) p_i^{\,-2\sigma}(\ldots)^{-2} M_x$$

$$p_i \frac{\partial X_i}{\partial p_i} = -\sigma p_i^{\,-\sigma}(\ldots)^{-1} M_x + (\sigma-1) p_i^{\,-2\sigma+1}(\ldots)^{-2} M_x$$

$$\frac{p_i}{X_i}\frac{\partial X_i}{\partial p_i} = -\sigma + (\sigma-1) p_i^{\,-\sigma+1}(\ldots)^{-1} = -\sigma + s_i(\sigma-1)$$

A convention is to define the Marshallian price elasticity as positive

$$\eta \equiv -\frac{p_i}{X_i}\frac{\partial X_i}{\partial p_i} = \sigma - (\sigma - 1)p_i^{-\sigma + 1}(...)^{-1} = \sigma - s_i(\sigma - 1)$$

7.2 Monopoly with fixed costs

Now suppose that we have a two-sector economy, X and Y, where the representative consumer has CES preferences between the two goods.

X is produced with increasing returns to scale in the form of a fixed costs plus a constant marginal cost in terms of the single factor of production, labor (L).

We assume in this section that there is a single monopoly producer of X who must incur the fixed costs (FC). Agent ENTR receives markup revenues and demands fixed costs.

Suppose demand for good X is just written in inverse form p(X) so the monopolist’s revenue is R = p(X)X. Marginal revenue is then given by:

(1)

$$\frac{\partial R}{\partial X} = p + X\frac{\partial p}{\partial X} = p + p\left[\frac{X}{p}\frac{\partial p}{\partial X}\right] = p\left[1 - \frac{1}{\eta}\right] \equiv MR$$

$$MR = p(1 - mk) \qquad mk = \frac{1}{\eta} \qquad \eta \equiv -\left[\frac{p}{X}\frac{\partial X}{\partial p}\right]$$

where mk is the optimal markup.

η is the Marshallian elasticity of demand, defined as positive. The monopoly markup is just the inverse of this elasticity.

Let's calibrate under the assumption that this is a "natural monopoly": the profit-maximizing entrepreneur just breaks even.

Markets	C	FC	Y	W	CONS	ENTR
	Production Sectors				Consumers	
PX	100			-100		
PY			100	-100		
PFC		20				-20
PW				200	-200	
PL	-80	-20	-100		200	
MK	-20					20

Choose units so that the price of X and the marginal cost of X = 1.

Then 1*(1 - mk) = 0.8, so mk = 0.2. The observed expenditure share on X is s = 0.5 in the benchmark.

$mk = 1/[\sigma - s(\sigma-1)] = 0.8$, so $[\sigma - s(\sigma-1)] = [\sigma - 0.5*(\sigma-1)] = 5$

implies $\sigma = 9$ $(9 - 0.5*8) = 5$

Two unknowns are added to a standard competitive model:

```
SHAREX  Share of X in consumption (value share)
MARKUP  Markup;
```

And two equations (where sigma is a parameter = 9).:

```
SHX.. SHAREX =E= 100*PX*X /(100*PX*X + 100*PY*Y);
MK..  MARKUP =E= 1/(SIGMA - (SIGMA-1)*SHAREX);
```

We could break out the entrepreneur (whoever has the property rights to the income stream) as a separate consumer.

The problem with doing this is if profits are negative, then the model will not solve.

Suppose that the entrepreneur has the same preferences as everyone else.

Then we will just calculate aggregate income and break out monopoly profits after solving.

```
INCOME.. CONS =E= PL*ENDOWL + (100*PX*X*MARKUP - PL*FC);
```

```
$TITLE: M7-2.GMS:  Monopoly with fixed costs

$ONTEXT
                        Production Sectors            Consumers
  Markets    |    C      FC        Y         W     |  CONS    ENTR
--------------------------------------------------------------
       PX    |  100                        -100    |
       PY    |                    100      -100    |
       PFC   |          20                         |            -20
       PW    |                              200    |  -200
       PL    |  -80    -20       -100              |   200
       MK    |  -20                                |             20

$OFFTEXT

PARAMETERS
  SIGMA      SIGMA: elasticity of substitution among varieties
  FC         parameter setting the level of fixed costs
  ENDOWL     endowment of labor
  INCOMEM    monopoly profit share (markup revenues - fixed costs)
  INCOMEC    income share of the "the people"
  MODELSTAT  statistic indicating model solved: 0 = solved;

SIGMA = 9;
FC = 20;
ENDOWL = 200;
```

```
POSITIVE VARIABLES

  X       Activity level for X (output per firm)
  Y       Activity level of Y output
  W       Activity level for welfare

  PX      Price of X
  PY      Price of Y
  PW      Price index for utility (consumer price index)
  PL      Price of labor

  CONS      Income of the representative consumer
  SHAREX  Share of X in consumption (value share)
  MARKUP  Markup;

EQUATIONS

  PRICEX     MR = MC in X (associated with X output per firm)
  PRICEY     Zero profit condition for Y (PY = MC)
  PRICEW     Zero profit condition for W (PW = MC of utility)

  MKT_X      Supply-demand balance for X (individual variety)
  MKT_Y      Supply-demand balance for Y
  MKT_W      Supply-demand balance for utility W (welfare)
  MKT_L      Supply-demand balance for labor
```

```
  INCOME     National income
  SHX        Share of X in expenditure
  MK         Markup equation;

PRICEX..    80*PL =G= 100*PX*(1-MARKUP);

PRICEY..    100*PL =G= 100*PY;

PRICEW..    (0.5*PX**(1-SIGMA) + 0.5*PY**(1-SIGMA))**(1/(1-SIGMA))
                 =G= PW;

MKT_X..     X*100  =G= PX**(-SIGMA)*(PW**(SIGMA-1))*CONS/2;

MKT_Y..     Y*100 =G=  PY**(-SIGMA)*(PW**(SIGMA-1))*CONS/2;

MKT_W..     200*W =G=  CONS/PW;

MKT_L..     ENDOWL =E= Y*100 + X*80 + FC;

INCOME..    CONS =E= PL*ENDOWL + (100*PX*X*MARKUP - PL*FC);

SHX..       SHAREX =E= 100*PX*X / (100*PX*X + 100*PY*Y) ;

MK..        MARKUP =E= 1/(SIGMA - (SIGMA-1)*SHAREX);
```

```
MODEL MONOPOLY /PRICEX.X, PRICEY.Y, PRICEW.W,
                MKT_X.PX, MKT_Y.PY, MKT_W.PW, MKT_L.PL,
                INCOME.CONS, SHX.SHAREX, MK.MARKUP/;

OPTION MCP=PATH;

*       set benchmark values:
X.L = 1;
Y.L = 1;
W.L = 1;
PX.L = 1;
PY.L = 1;
PL.L = 1;
PW.L = 1;
CONS.L = 200;
SHAREX.L = 0.5;
MARKUP.L = 0.20;

* choose the price of good Y as numeraire

PY.FX = 1;

* check for calibration and starting-value errors

MONOPOLY.ITERLIM = 0;
SOLVE MONOPOLY USING MCP;
```

```
MONOPOLY.ITERLIM = 1000;
SOLVE MONOPOLY USING MCP;

MODELSTAT = MONOPOLY.MODELSTAT - 1.;
DISPLAY MODELSTAT;

INCOMEM = (MARKUP.L*PX.L*X.L*100 - PL.L*FC)/CONS.L;
INCOMEC = (PL.L*ENDOWL)/CONS.L;

DISPLAY INCOMEM, INCOMEC;

* Counterfactual: contract the size of the economy

ENDOWL = 100;

SOLVE MONOPOLY USING MCP;

INCOMEM = (MARKUP.L*PX.L*X.L*100 - PL.L*FC)/CONS.L;
INCOMEC = (PL.L*ENDOWL)/CONS.L;

DISPLAY INCOMEM, INCOMEC;

* Counterfactual: expand the size of the economy

ENDOWL = 400;
```

```
SOLVE MONOPOLY USING MCP;

INCOMEM = (MARKUP.L*PX.L*X.L*100 - PL.L*FC)/CONS.L;
INCOMEC = (PL.L*ENDOWL)/CONS.L;

DISPLAY INCOMEM, INCOMEC;
```

7.3 Oligopoly: Cournot competition with identical products and free entry

Model will be characterized by variable markups and pro-competitive gains from trade.

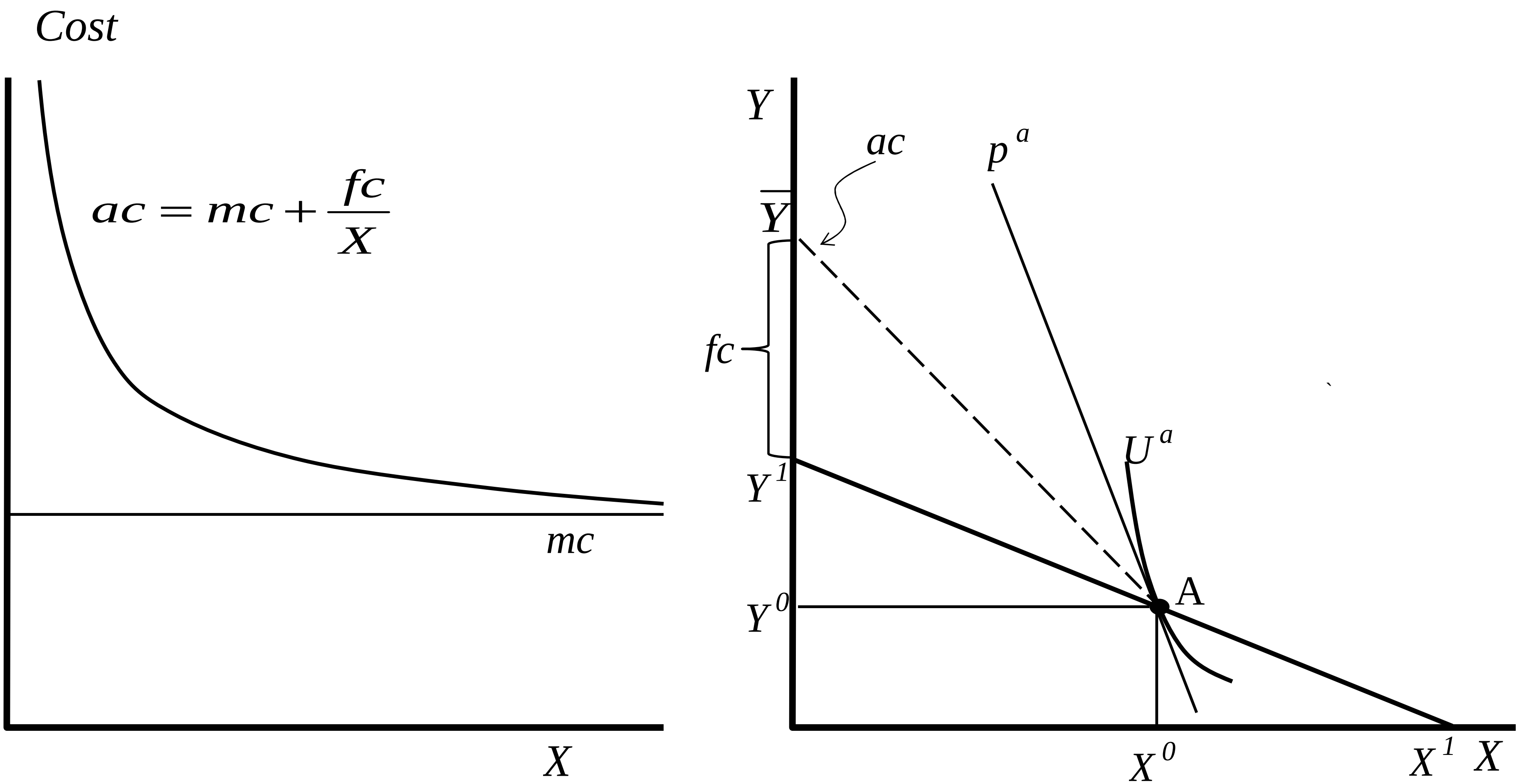
A single factor of production *L* (call it labor) divided between the *Y* and *X* sectors and among firms in the *X* sector.

Marginal cost in units of labor is denoted by *mc* and total cost (*tc*) and average cost (*ac*) for an *X* firm are as follows:

$$tc = cX + f \qquad ac = \frac{tc}{X} = c + \frac{f}{X}$$

General equilibrium production frontier shown in Figure 2.

Figure 11.1



The average cost of producing *X* at point A is given by the labor needed for *X* divided by the output of *X.*

$$ac = \frac{\bar{L} - L^0}{X^0} = \frac{\bar{Y} - Y^0}{X^0}$$

Too little *X* is produced at too high a price.

Cournot-Nash (or Cournot for short) competition in which firms pick a quantity as a best response to their rivals' quantities.

Revenue for a Cournot firm i and selling in country j is given by the price in j times quantity of the firm's sales. Price is a function of all firms' sales.

$R_{ij} = p_j(X_j)X_{ij}$. X_j is total sales in market j: $X_j = \sum_i X_{ij}$

Cournot conjectures imply that $\partial X_j / \partial X_{ij} = 1$; a one-unit increase in the firm's own supply is a one-unit increase in market supply.

Marginal revenue is then

$$\frac{\partial R_{ij}}{\partial X_{ij}} = p_j + X_{ij}\frac{\partial p_j}{\partial X_j}\frac{\partial X_j}{\partial X_{ij}} = p_j + X_{ij}\frac{\partial p_j}{\partial X_j} \qquad \text{since } \frac{\partial X_j}{\partial X_{ij}} = 1$$

Now multiple and divide the right-hand equation by total market supply and also by the price.

$$\frac{\partial R_{ij}}{\partial X_{ij}} = p_j + X_{ij}\frac{\partial p_j}{\partial X_j} = p_j + p_j\frac{X_{ij}}{X_j}\left[\frac{X_j}{p_j}\frac{\partial p_j}{\partial X_j}\right]$$

The term in square brackets in is just the inverse of the price elasticity of demand.

$$\frac{\partial R_{ij}}{\partial X_{ij}} = p_j\left[1 - \frac{X_{ij}}{X_j}\frac{1}{\eta_j}\right] \qquad \eta_j \equiv -\left[\frac{p_j}{X_j}\frac{\partial X_j}{\partial p_j}\right] \qquad (elast\ of\ demand)$$

The term X_{ij}/X_j in (11.6) is just firm i's market share in market j, which we can denote by s_{ij} .

$$mr_{ij} = p\left[1 - \frac{s_{ij}}{\eta_j}\right] = mc_i$$

$$U = (nX)^{\alpha} Y^{1-\alpha}$$ with income $$I = \bar{L} + \Pi = pnX + Y$$

Let the price of *Y* be numeraire, equal to one.

$$nX = \frac{\alpha I}{p} \qquad Y = (1-\alpha)I$$

The elasticity of demand for *X*, we will find that η = 1. The markup is just the firm's market share, which in turn is just 1/*n*

$$p(1 - 1/n) = c \qquad p = \frac{n}{n-1}c$$

Consider the free entry and exit version of the Cobb-Douglas case.

$$p(1 - 1/n) = c \qquad \text{pricing for X} \qquad (1)$$

Free entry or zero-profits

$$pX = cX + f \quad => \quad (p/n)X = f$$ pricing for n

(markups revenues equal fixed costs)

$$nX = \alpha \bar{L}/p$$ market clearing

Use the first two equations:

$$\frac{n}{n-1} = 1 + \frac{f}{cX} \quad \Rightarrow \quad \frac{n}{n-1} - \frac{n-1}{n-1} = \frac{f}{cX}$$

which gives us output per firm.

$$X = (n-1)\frac{f}{c} \tag{2}$$

Multiple both sides of pricing equation (1) by *X*, and substitute for *pX* from market clearing.

$$p\left(1 - \frac{1}{n}\right)X = p\left(\frac{n-1}{n}\right)X = \left(\frac{n-1}{n}\right)\frac{\alpha \bar{L}}{n} = cX \tag{3}$$

Now substitute the expression for *X* in (2) to give us a solution for *n*, the endogenous number of firms.

$$\left(\frac{n-1}{n}\right)\frac{\alpha \bar{L}}{n} = c(n-1)\frac{f}{c} \qquad n^2 = \frac{\alpha \bar{L}}{f} \tag{4}$$

Take the square root of the right-hand equation to get n and then substitute this into to get *X.*

$$n = \sqrt{\frac{\alpha \bar{L}}{f}} \qquad X = \left[\sqrt{\frac{\alpha \bar{L}}{f}} - 1\right]\frac{f}{c} \qquad (5)$$

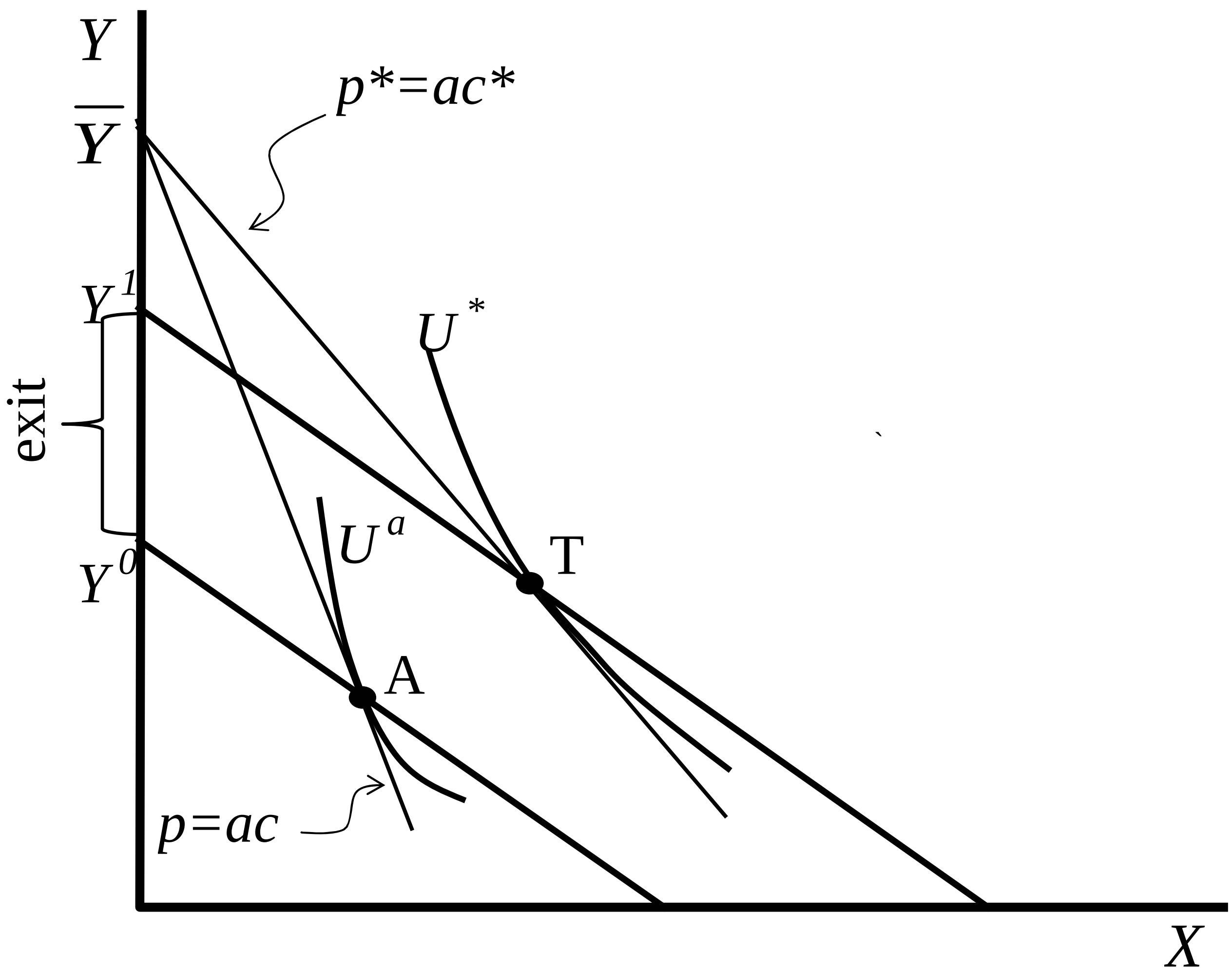
$$markup = \sqrt{\frac{f}{\alpha \bar{L}}}$$

Pro-competitive, pro-efficiency gains from trade: a larger market supports more competition and more output per firm

In the model, we use another feature which is not needed, but allows us to keep track of markup revenues relative to total income.

We introduce a (dummy) agent called “ENTRE” for entrepreneur, who received the markup revenues and demands fixed costs.

Figure 11.4



	Production Sectors				Consumers	
Markets	C	FC	Y	W	CONS	ENTR
PX	100			-100		
PY			100	-100		
PN		20				-20
PW				200	-200	
PL	-80	-20	-100		200	
MK	-20					20

This is a way of capturing free entry: in equilibrium, markup revenues are exactly exhausted in paying for fixed costs.

In the data for the model, we assume that the fixed costs of 20 are the combined fixed costs of 5 firms initially in the market.

If preferences are Cobb-Douglas, then the initial markup is 1/N = 1/5 = 0.20 which is consistent with the initial data.

Then in the calibration, the initial value of N is N.L = 5. The marginal cost of X is 1, so the price of X must be PX.L = 1.25.

```
DN..      N*4 =G= ENTRE/PN;                 demand for fixed costs

IENTRE.. ENTRE =E= MARKUP*PX*X*80;          total markup revenues

MK..      MARKUP*N =E= 1;                   markup formula
```

where MARKUP is an endogenous auxiliary variable.

```
$TITLE  M7-3.GMS: Oligopoly with Free Entry, homogeneous good, Cournot
* competition.  Uses Cobb-Douglas demand

$ONTEXT
                  Production Sectors                  Consumers
 Markets    |   X        N        Y        W     |  CONS   ENTRE
 ----------------------------------------------------------
      PX    | 100                          -100   |
      PY    |                   100        -100   |
      PN    |           20                        |          -20
      PW    |                               200   | -200
      PL    |  80      -20     -100               |  200

      MK    | -20                                 |           20

$OFFTEXT

PARAMETERS
 SIGMA      Elasticity of substitution
 ENDOW      Endowment scale multiplier
 MODELSTAT  statistic indicating model solved: 0 = solved
 XPF        X output per firm;

SIGMA = 1;
ENDOW = 200;
```

```
POSITIVE VARIABLES

  X          Aggregate X production by all firms
  N          Number of X sector firms
  Y          Activity level of Y output
  W          Activity level for welfare

  PX         Price of an individual X variety
  PN         Price of fixed costs (price of entering)
  PY         Price of Y
  PW         Price index for utility (consumer price index)

  PL         Price of labor


  CONS       Income of the representative consumer
  ENTRE      Inomce of the agent ENTRE = markup revenue
  MARKUP     Endogenous markup rate = 1 over N;


EQUATIONS

  PRICEX      MR = MC in X
  PRICEN      Zero profit condition for fixed costs
```

```
  PRICEY      Zero profit condition for Y (PY = MC)
  PRICEW      Zero profit condition for W

  DX          Supply-Demand for X
  DN          Supply-Demand for fixed costs
  DY          Supply-Demand for Y
  DW          Supply-Demand for W

  LAB         Supply-demand balance for labor

  ICONS       Consumer (factor owners') income
  IENTRE      Entrepreneur's profits
  MK          Markup equation;


PRICEX..   PL =G= PX*(1 - MARKUP);

PRICEN..   PL =G= PN;

PRICEY..   PL =G= PY;

PRICEW..   ((PX/1.25)**0.5)*(PY**0.5) =G= PW;

DX..       X*80 =E= 0.5*CONS/PX;

DN..       N*4 =G= ENTRE/PN;
```

```
DY..         Y*100 =E= 0.5*CONS/PY;

DW..         W*200 =E= CONS/PW;

LAB..        ENDOW =E= Y*100 + X*80 + N*4;

ICONS..      CONS =E= PL*ENDOW;

IENTRE..     ENTRE =E= MARKUP*PX*X*80;

MK..         MARKUP*N =E= 1;


MODEL M52 /DX.PX, DY.PY, DW.PW, DN.PN, PRICEX.X, PRICEY.Y,
           PRICEW.W, PRICEN.N,LAB.PL,
           ICONS.CONS, IENTRE.ENTRE, MK.MARKUP/;

OPTION MCP=MILES;
OPTION LIMROW=0;
OPTION LIMCOL=0;
$OFFSYMLIST OFFSYMXREF OFFUELLIST OFFUELXREF

CONS.L = 200;
X.L = 1;
Y.L = 1;
```

```
W.L = 1;
N.L = 5;
PX.L = 1.25;
PY.L = 1;
PL.L = 1;
PW.L = 1;
PN.L = 1;
ENTRE.L = 20;
MARKUP.L = 0.20;

PY.FX = 1;

M52.ITERLIM = 0;
SOLVE M52 USING MCP;
MODELSTAT = M52.MODELSTAT - 1.;

M52.ITERLIM = 1000;
SOLVE M52 USING MCP;
MODELSTAT = M52.MODELSTAT - 1.;

XPF = 80*X.L/N.L;
DISPLAY XPF;

* counterfactual: double the size of the economy

ENDOW = 400;
```

```
SOLVE M52 USING MCP;

XPF = 80*X.L/N.L;
DISPLAY XPF;


* show welfare as a function of the economy's size

SETS I indexes 25 different size levels  /I1*I25/;

PARAMETERS
 SIZE(I)
 WELFARE(I)
 WELFCAP(I)
 FIRMSIZE(I)
 FIRMNUMB(I)
 MARKUPO(I)
 RESULTS(I,*);

LOOP(I,

SIZE(I) =  5.2 - 0.2*ORD(I);
ENDOW = 200*SIZE(I);

SOLVE M52 USING MCP;
```

```
WELFARE(I) = W.L;
WELFCAP(I) = WELFARE(I)/SIZE(I);
FIRMSIZE(I) = X.L/N.L*5;
FIRMNUMB(I) = N.L/5;
MARKUPO(I) = MARKUP.L;

);

RESULTS(I, "SIZE") = SIZE(I);
RESULTS(I, "WELFARE") = WELFARE(I);
RESULTS(I, "WELFCAP") = WELFCAP(I);
RESULTS(I, "FIRMSIZE") = FIRMSIZE(I);
RESULTS(I, "FIRMNUMB") = FIRMNUMB(I);
RESULTS(I, "MARKUP")   = MARKUPO(I);

DISPLAY RESULTS;

*  Write parameter RESULTS to an Excel file M7.XLS,
*  starting in Sheet1,

$LIBINCLUDE XLDUMP RESULTS M7.XLS SHEET1!A3


Execute_Unload 'M7.gdx' RESULTS
execute 'gdxxrw.exe M7.gdx par=RESULTS rng=SHEET2!A3'
```

7.4 Monopolistic-competition I: large group with D-S CES

The assumption in "large-group" monopolistic competition is that there are many firms: individual firms view e, M as *constants*.

Thus the elasticity of demand for an individual variety is just σ.

Equilibrium in the X sector involves two equations in two unknowns. The unknowns are X, output per variety and N, the numbers of varieties or firms.

The two equations are the firm's optimization condition, marginal revenue equals marginal cost, and the free-entry or zero profit condition, prices equals average cost.

Gains from increased final and intermediate goods variety.

Total income is given by *L* when the wage is chosen as numeraire.

Symmetry I: all X goods are imperfect but symmetric substitutes

Symmetry I: all X goods have the same cost function

Symmetry III: fixed and marginal costs have the same functional form: f/c is a constant.

X and p_x will denote the price of a representative good which are the same for all goods actually produced

$$U = \left[\sum_i X_i^{\alpha}\right]^{\frac{\beta}{\alpha}} Y^{1-\beta} \qquad \sigma = \frac{1}{1-\alpha} \qquad L = np_x X + p_y Y \qquad (1)$$

the consumer’s demands for *X* varieties and *Y* are

$$Y = (1 - \beta)\frac{L}{p_y} \qquad X_i = p_{xi}^{-\sigma}\left[\sum_i p_{xi}^{1-\sigma}\right]^{-1}\beta L \qquad nX = \beta\frac{L}{p_x} \qquad (2)$$

The variety’s own price appears both as the first term on the right-hand side of the second equation of (2) but also appears in the summation term inside the square brackets.

The effect of a change in a firm’s price on the summation term in square brackets become extremely small as the number of varieties (firms) n becomes large.

Assumes that an individual firm is too small to affect the summation term in (2), an assumption known as “large-group monopolistic competition.

The price elasticity of demand for an individual good is given simply by σ, the elasticity of substitution among the *X* goods

$$-\frac{p_x}{X}\frac{\partial X}{\partial p_x} = \sigma \qquad mr_x = p_x(1 - 1/\sigma) = mc_x \qquad (3)$$

Inequality	Definition	Complement Var	
$p_x(1 - 1/\sigma) \le mc_x$	pricing for *X*	*X*	(4)
$(p_x/\sigma)X \le fc_x$	pricing for *n* (free entry)	*n*	(5)
$p_y \le mc_y$	pricing for *Y*	*Y*	(6)

Then there are three market-clearing conditions, which require that supply equal demand (strictly speaking supply is greater than or equal to demand)

$$(1-\beta)L/p_y \leq Y \qquad \text{demand/supply } Y \qquad p_y \qquad (7)$$

$$\beta L/p_x \leq nX \qquad \text{demand/supply } X \text{ varieties} \qquad p_x \qquad (8)$$

$$(mc_y)Y + n(mc_x)X + n(fc_x) = L \qquad \text{demand/supply } L \qquad w \qquad (9)$$

Equations (4) and (5) can be solved for both X and p_x. Then these solution values can be used in (8) to get n.

$$X = (\sigma - 1)\frac{fc_x}{mc_x} \qquad n = \frac{\beta L}{\sigma fc_x} \qquad nX = \frac{(\sigma - 1)}{\sigma}\frac{\beta L}{mc_x} \qquad (10)$$

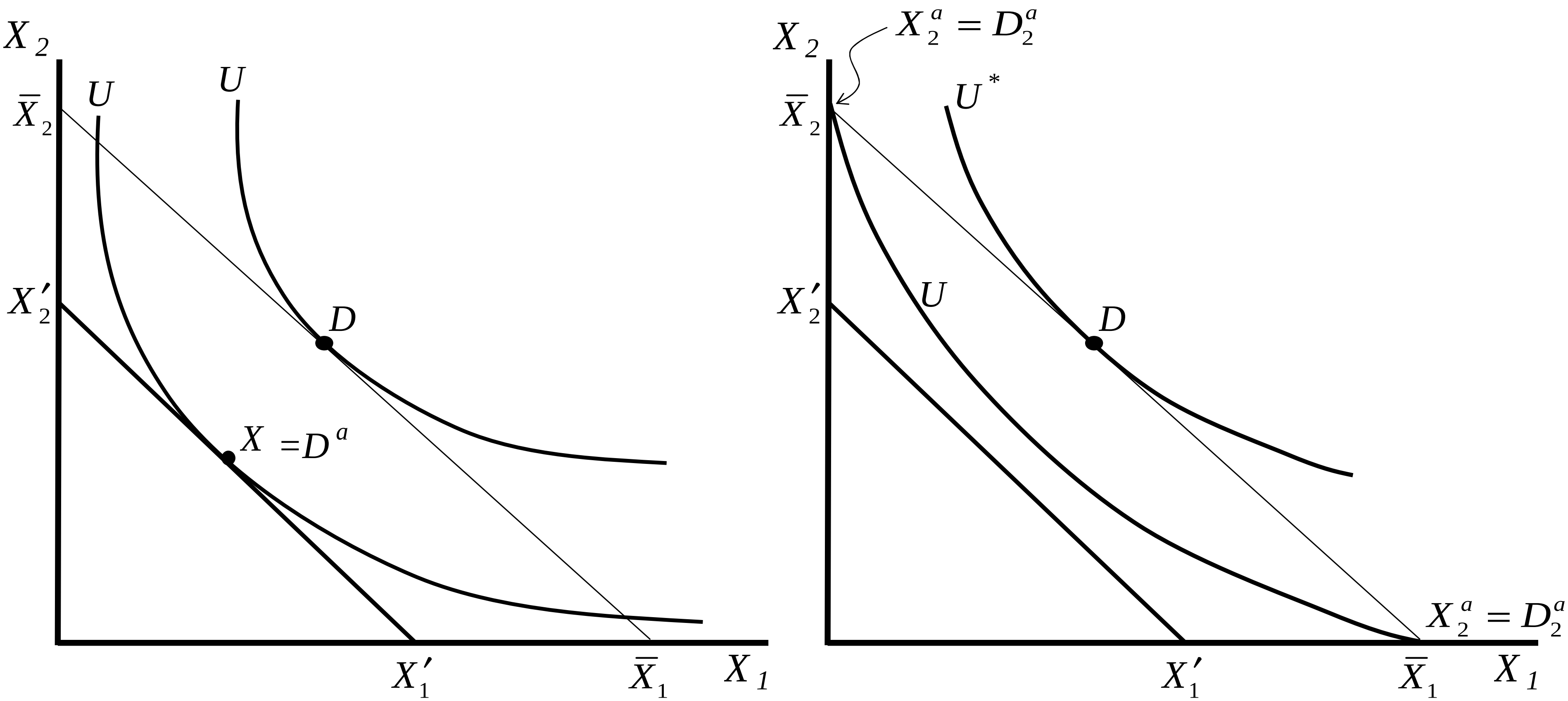
The output of any good that is produced is a constant and that any expansion in the economy creates a proportional increases in variety n.

Let *X/L*, the consumption of a representative variety per capita, be given by *C*. Then note from the last equation of (10) that *nC* is a constant:

$$C = \frac{X}{L} = \frac{(\sigma - 1)}{\sigma} \frac{\beta}{mc_x} \frac{1}{n} \equiv \frac{\gamma}{n} \qquad (11)$$

$$U_x = \left[nC^{\alpha} \right]^{\frac{1}{\alpha}} = n^{\frac{1}{\alpha}} C = n^{\frac{1-\alpha}{\alpha}} \gamma = n^{\frac{1}{\sigma - 1}} \gamma = \left[\frac{\beta L}{\sigma f c_x} \right]^{\frac{1}{\sigma - 1}} \gamma$$

Figure 12.1



X_2
$\bar{X}_2$
U
U
X_2'
D
$X = D^a$
X_1'
$\bar{X}_1$
X_1
$X_2^a = D_2^a$
X_2
$\bar{X}_2$
U^*
X_2'
U
D
$X_2^a = D_2^a$
X_1'
$\bar{X}_1$
X_1

	Production Sectors				Consumers	
Markets	C	FC	Y	W	CONS	ENTR
PX	100			-100		
PY			100	-100		
PN		20				-20
PW				200	-200	
PL	-80	-20	-100		200	
MK	-20					20

There are a number of ways to organize the benchmark data, this is one of them.

Markup revenues (MK) are not directly observed by IO economists have techniques for estimating these.

I introduce an artificial or "dummy" agent ENTR (entrepreneur). ENTR receives the markup revenues and "demands" fixed costs.

In equilibrium, the total value of fixed costs produced equals markup revenues, which is a way of modeling the free-entry zero-profit condition.

The activity level for N (production of fixed costs) corresponds to the number of varieties produced in equilibrium, and so affects the price index and welfare.

marginal revenue = mc   price = average cost

$$p(1 - 1/\sigma) = p(1 - mk) = mc \qquad p = mc + fc/X$$

Subtracting the second equation from the first:

$$p(1 - mk) - p = mc - mc - fc/X$$

$$p(mk)X = fc$$ markup revenues = fixed costs.

The counter-factual experiment doubles the size of the economy.

The X sector's output is homogeneous of degree 1.25 in factor inputs with σ = 5, if by X sector's output here we mean X_c.

The X sector expands only through the entry of new firms, the output of a representative firm, X, is constant. X_c is given by

$$X_c = \left[NX^{\alpha}\right]^{\frac{1}{\alpha}} = N^{\frac{1}{\alpha}}X = N^{\frac{\sigma}{\sigma - 1}}X = N^{1.25}X$$

```
$TITLE: M7-4.GMS:  Large-Group Monopolistic Competition
* calibrated to an elasticity of substitution of 5

$ONTEXT
                        Production Sectors            Consumers
  Markets   |   XC       N         Y         W     |  CONS    ENTR
-------------------------------------------------------------------
       PX   |  100                         -100    |
       PY   |                    100       -100    |
       PN   |           20                         |          -20
       PW   |                               200    |  -200
       PL   |  -80     -20      -100               |   200

       MK   |  -20                                 |           20

$OFFTEXT


PARAMETERS
  SI         SIGMA: elasticity of substitution among varieties
  FC         parameter setting the level of fixed costs
  ENDOWL     endowment of labor
  MODELSTAT  statistic indicating model solved: 0 = solved;

SI = 5;
FC = 20;
```

```
ENDOWL = 200;

POSITIVE VARIABLES

  X      Activity level for X (output per firm)
  XC     Composite X (utility value of agg X sector output)
  N      Number of X sector firms (variety measure)
  Y      Activity level of Y output
  W      Activity level for welfare

  PX     Price of an individual X variety
  PE     Price index (unit expenditure function): cost of XC = 1
  PN     Price of fixed costs (price of entering)
  PY     Price of Y
  PW     Price index for utility (consumer price index)

  PL     Price of labor

  CONS   Income of the representative consumer;

EQUATIONS

  PRICEX     MR = MC in X (associated with X output per firm)
  PINDEX     Price index for X sector goods
  PRICEN     Zero profits - free entry in X (associated with N)
  PRICEY     Zero profit condition for Y (PY = MC)
```

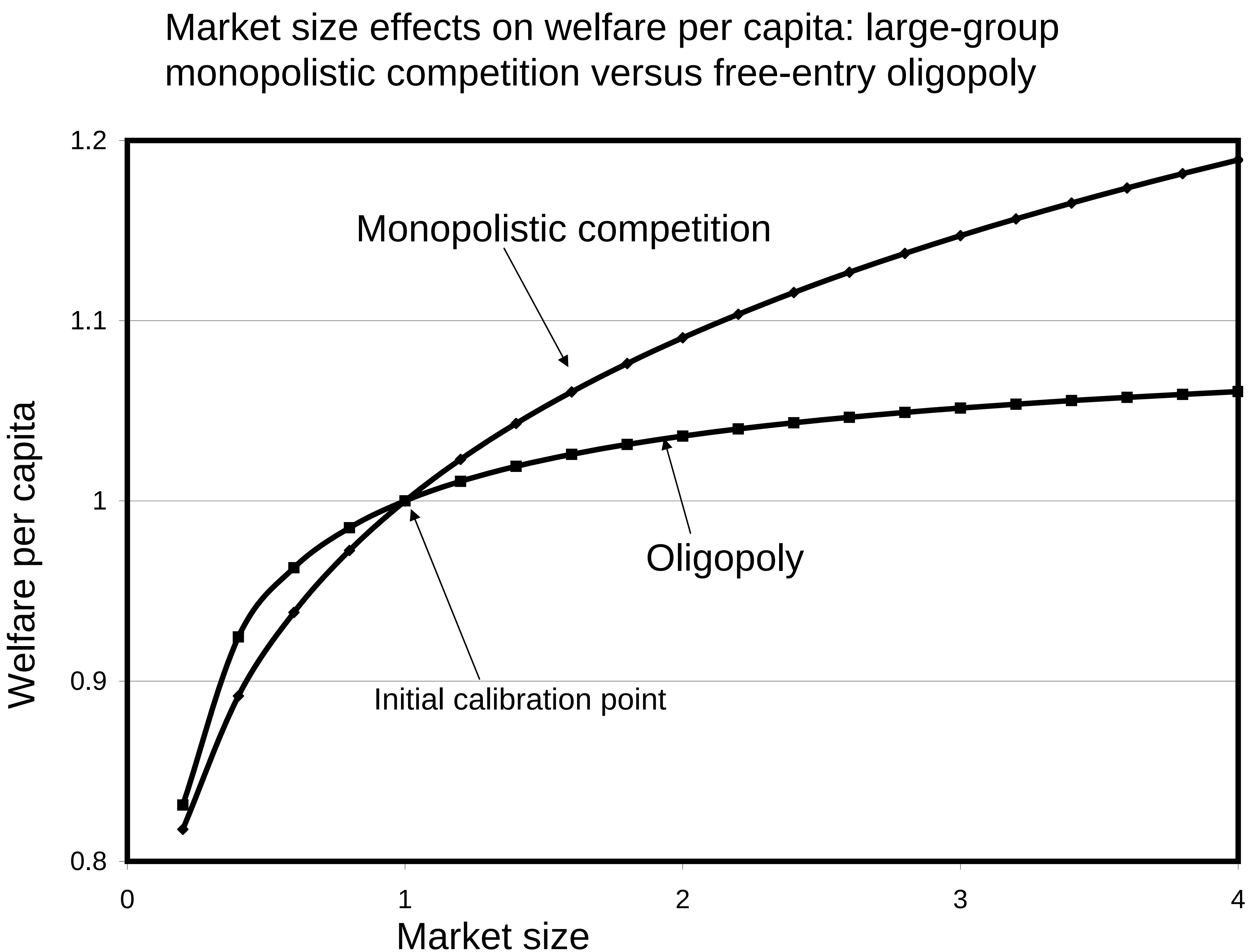
```
  PRICEW     Zero profit condition for W (PW = MC of utility)

  DX         Supply-demand balance for X (individual variety)
  DXC        Supply-demand balance for XC
  DN         Supply-demand for firms N: markup rev = fexed cost
  DY         Supply-demand balance for Y
  DW         Supply-demand balance for utility W (welfare)

  LAB        Supply-demand balance for labor

  INCOME     National income;


PRICEX..   PL =G= PX*(1-1/SI);

PINDEX..   (N*PX**(1-SI))**(1/(1-SI)) =G= PE;

PRICEN..   PL =G= PN;

PRICEY..   PL =G= PY;

PRICEW..   (PE**0.5)*(PY**0.5) =G= PW;


DX..       X*80  =G=  PX**(-SI)*(PE**(SI-1))*CONS/2;
```

```
DXC..       XC =G= N**(SI/(SI-1))*X;

DN..        N*FC  =G= (PX/SI)*X*80*N/PN;

DY..        Y*100 =G= CONS/(2*PY);

DW..        200*W =G= (1.25**0.5)*CONS/PW;

LAB..       ENDOWL =E= Y*100 + N*X*80 + N*FC;

INCOME..    CONS =E= PL*ENDOWL;


MODEL M62 /PRICEX.X, PRICEY.Y, PRICEW.W, PRICEN.N, PINDEX.XC,
           DX.PX, DXC.PE, DN.PN, DY.PY, DW.PW,
           LAB.PL, INCOME.CONS/;


*       set benchmark values:

PE.L = 1.25;
CONS.L = 200;
X.L = 1;
XC.L = 1;
Y.L = 1;
N.L = 1;
```

```
W.L = 1;
PX.L = 1.25;
PN.L = 1;
PY.L = 1;

PL.L = 1;
PW.L = 1.25**0.5;

* choose the price of good Y as numeraire

PY.FX = 1;

* check for calibration and starting-value errors

M62.ITERLIM = 0;
SOLVE M62 USING MCP;

M62.ITERLIM = 1000;
SOLVE M62 USING MCP;

MODELSTAT = M62.MODELSTAT - 1.;

DISPLAY MODELSTAT;

* Counterfactual: expand the size of the economy
```

```
ENDOWL = 400;

SOLVE M62 USING MCP;



* show welfare as a function of the economy's size

SETS I indexes 25 different size levels  /I1*I25/;

PARAMETERS
 SIZE(I)
 WELFARE(I)
 WELFCAP(I)
 FIRMSIZE(I)
 FIRMNUMB(I)
 MARKUPM(I)
 RESULTS(I,*);

LOOP(I,

SIZE(I) =  5.2 - 0.2*ORD(I);
ENDOWL = 200*SIZE(I);

SOLVE M62 USING MCP;
```

```
WELFARE(I) = W.L;
WELFCAP(I) = WELFARE(I)/SIZE(I);
FIRMSIZE(I) = X.L;
FIRMNUMB(I) = N.L;
MARKUPM(I) = 1/SI;

);

RESULTS(I, "SIZE")    = SIZE(I);
RESULTS(I, "WELFARE") = WELFARE(I);
RESULTS(I, "WELFCAP") = WELFCAP(I);
RESULTS(I, "FIRMSIZE") = FIRMSIZE(I);
RESULTS(I, "FIRMNUMB") = FIRMNUMB(I);
RESULTS(I, "MARKUP")   = MARKUPM(I);

DISPLAY RESULTS;

* Write parameter RESULTS to an Excel file TRCOST.XLS,
* starting in Sheet1,

$LIBINCLUDE XLDUMP RESULTS M7.XLS SHEET1!A31


Execute_Unload 'M7.gdx' RESULTS
execute 'gdxxrw.exe M7.gdx par=RESULTS rng=SHEET2!A31'
```



Market size effects on welfare per capita: large-group
monopolistic competition versus free-entry oligopoly
Monopolistic competition
Oligopoly
Initial calibration point
Welfare per capita
Market size
1.2
1.1
1
0.9
0.8
0
1
2
3
4

7.5 Monopolistic-competition II: small group

Small-group monopolistic competition. Firms are Bertrand competitors, choosing their price holding the prices of the other firms constant.

The demand for an individual variety i:

$$X_i = p_i^{-\sigma}\left[\sum p_j^{1-\sigma}\right]^{-1} M_x \qquad \sigma = \frac{1}{1-\alpha}, \qquad \alpha = \frac{\sigma - 1}{\sigma}$$

Large-group monopolistic competition assumes that the number of firms is large so that each firm views the term in brackets as a constant. But this is an approximation.

$$\eta_i = -\frac{p_i}{X_i}\frac{\partial X_i}{\partial p_i} = \sigma - s_i(\sigma - 1) \qquad s_i = \frac{p_i X_i}{M_x} = \frac{p_i^{1-\sigma}}{\left[\sum p_j^{1-\sigma}\right]}$$

where s_i is the share of X-sector expenditure on good i.

If all firms are identical (e.g., domestic firms that have the same price).

$$\eta_i = \sigma - \frac{1}{N}(\sigma - 1) \qquad markup = \frac{1}{\sigma - \frac{1}{N}(\sigma - 1)}$$

However, we will have some problem comparing this to our large-group monopolistic-competition model.

This new one would require re-calibrating the whole model, but then we would have a different elasticity of substitution.

An alternative is to use a simple parameter, a “cluge” or “fudge factor” that makes the two consistent.

In our case with N = 1 initially, we choose a constant 0.20 = 1/σ so that the same benchmark data fits the small-group model.

$$markup = \frac{0.20}{\sigma - \frac{1}{N}(\sigma - 1)} = \frac{N/\sigma}{N\sigma - (\sigma - 1)}$$

Now the large-group and small-group versions will both reproduce the benchmark data with N = 1.

There is a problem with this formulation, which occurs when N < 1.

This arises due to the modeling of N as a continuous variable when in reality it is discrete and bound from below by 1.

A quick fix is to replace N in the markup formula with (N+1), and then adjust the “fudge” or “cluge” to offset this.

$$markup = \frac{0.60}{\sigma - \frac{1}{(N+1)}(\sigma - 1)}$$

Then η => 1 as N => 0.

This is the second version of the model shown below, and calibrates to the same initial markup, 0.20, as before.

```
$TITLE: M7-5.GMS: Small-Group Monopolistic Competition
* markup formula is 1/(sigma - (1/(1+N))(sigma - 1)
* to calibrate to the same data, sigma = 5, N = 1, a fudge-factor
* of 0.6 is used in the markup formula to reproduce the benchmark
* markup = 0.6/(sigma - (1/(1+N))(sigma - 1) = 0.20

$ONTEXT
                        Production Sectors           Consumers
  Markets   |   XC      N         Y         W     |  CONS    ENTR
-------------------------------------------------------------
       PX   |  100                       -100     |
       PY   |                   100      -100     |
       PN   |          20                         |          -20
       PW   |                             200     |  -200
       PL   |  -80    -20      -100               |   200

       MK   |  -20                                |           20

$OFFTEXT


PARAMETERS
  SI          SIGMA: elasticity of substitution among varieties
  FC          parameter setting the level of fixed costs
  ENDOWL      endowment of labor
  MODELSTAT   statistic indicating model solved: 0 = solved;
```

```
SI = 5;
FC = 20;
ENDOWL = 200;


NONNEGATIVE VARIABLES

  X     Activity level for X (output per firm)
  XC    Composite X (utility value of agg X sector output)
  N     Number of X sector firms (variety measure)
  Y     Activity level of Y output
  W     Activity level for welfare

  PX    Price of an individual X variety
  PE    Price index (unit expenditure function): cost of XC = 1
  PN    Price of fixed costs (price of entering)
  PY    Price of Y
  PW    Price index for utility (consumer price index)

  PL    Price of labor

  MK    Markup

  CONS  Income of the representative consumer;
```

```
EQUATIONS

  PRICEX    MR = MC in X (associated with X output per firm)
  PINDEX    Price index for X sector goods
  PRICEN    Zero profits - free entry in X (associated with N)
  PRICEY    Zero profit condition for Y (PY = MC)
  PRICEW    Zero profit condition for W (PW = MC of utility)

  DX        Supply-demand balance for X (individual variety)
  DXC       Supply-demand balance for XC
  DN        Supply-demand for firms N: markup rev = fexed cost
  DY        Supply-demand balance for Y
  DW        Supply-demand balance for utility W (welfare)

  LAB       Supply-demand balance for unskilled labor

  MARKUP    Markup equation

  INCOME    National income;


PRICEX..   PL =G= PX*(1 - MK);

PINDEX..   (N*PX**(1-SI))**(1/(1-SI)) =G= PE;

PRICEN..   PL =G= PN;
```

```
PRICEY..   PL =G= PY;

PRICEW..   (PE**0.5)*(PY**0.5) =G= PW;

DX..       X*80  =G=  PX**(-SI)*(PE**(SI-1))*CONS/2;

DXC..      XC =G= N**(SI/(SI-1))*X;

DN..       N*FC  =G= (PX*MK)*X*80*N/PN;

DY..       Y*100 =G= CONS/(2*PY);

DW..       200*W =G= (1.25**0.5)*CONS/PW;

LAB..      ENDOWL =E= Y*100 + N*X*80 + N*FC;

MARKUP..   MK =E= 0.6/(SI - 1/(N+1)*(SI - 1));

INCOME..   CONS =E= PL*ENDOWL;


MODEL M62 /PRICEX.X, PRICEY.Y, PRICEW.W, PRICEN.N, PINDEX.XC,
           DX.PX, DXC.PE, DN.PN, DY.PY, DW.PW,
           LAB.PL, MARKUP.MK, INCOME.CONS/;
```

```
*       set benchmark values:

PE.L = 1.25;
CONS.L = 200;
X.L = 1;
XC.L = 1;
Y.L = 1;
N.L = 1;
W.L = 1;
PX.L = 1.25;
PN.L = 1;
PY.L = 1;

PL.L = 1;
PW.L = 1.25**0.5;
MK.L = 0.20;
* choose the price of good Y as numeraire

PY.FX = 1;

* check for calibration and starting-value errors

M62.ITERLIM = 0;
SOLVE M62 USING MCP;

M62.ITERLIM = 1000;
```

```
SOLVE M62 USING MCP;

MODELSTAT = M62.MODELSTAT - 1.;

DISPLAY MODELSTAT;

* Counterfactual: expand the size of the economy

*ENDOWL = 400;

*SOLVE M62 USING MCP;



* show welfare as a function of the economy's size

SETS J scenario 1 = small-group mc 2 = large-group /J1*J2/;
SETS I indexes 25 different size levels  /I1*I25/;

PARAMETERS
 SIZE(I)
 WELFARE(I,J)
 WELFCAP(I,J)
 MARKUPS(I,J)
 NUMBERF(I,J)
 RESULTS(I,*);
```

```
MK.L = 0.2;

LOOP(I,
LOOP(J,

SIZE(I) =  5.2 - 0.2*ORD(I);
ENDOWL = 200*SIZE(I);

MK.UP = +INF;
MK.LO = 0;
MK.FX$(ORD(J) EQ 2) = 0.20;

SOLVE M62 USING MCP;

WELFARE(I,J) = W.L;
WELFCAP(I,J) = WELFARE(I,J)/SIZE(I);
MARKUPS(I,J) = MK.L;
NUMBERF(I,J) = N.L;

);
);

RESULTS(I, "SIZE") = SIZE(I);
RESULTS(I, "WELFCAP-L") = WELFCAP(I, "J2");
RESULTS(I, "WELFCAP-S") = WELFCAP(I, "J1");
```

```
RESULTS(I, "NUMBERF-L") = NUMBERF(I, "J2");
RESULTS(I, "NUMBERF-S") = NUMBERF(I, "J1");
RESULTS(I, "MARKUP-S")  = MARKUPS(I,"J1");


DISPLAY RESULTS;

* Write parameter RESULTS to an Excel file MCOMP2.XLS,
* starting in Sheet1, cell A3

Execute_Unload 'M7.gdx' RESULTS
execute 'gdxxrw.exe M7.gdx par=RESULTS rng=SHEET4!A3'
```

	SIZE	WELFCAP-L	WELFCAP-S	NUMBERF-L	NUMBERF-S	MARKUP-S
I1	5.00	1.22	1.21	5.00	3.63	0.145
I2	4.80	1.22	1.21	4.80	3.50	0.146
I3	4.60	1.21	1.20	4.60	3.38	0.147
I4	4.40	1.20	1.20	4.40	3.25	0.148
I5	4.20	1.20	1.19	4.20	3.13	0.149
I6	4.00	1.19	1.18	4.00	3.00	0.150
I7	3.80	1.18	1.18	3.80	2.87	0.151
I8	3.60	1.17	1.17	3.60	2.75	0.153
I9	3.40	1.17	1.16	3.40	2.62	0.154
I10	3.20	1.16	1.15	3.20	2.49	0.156
I11	3.00	1.15	1.14	3.00	2.36	0.157
I12	2.80	1.14	1.13	2.80	2.23	0.159
I13	2.60	1.13	1.12	2.60	2.10	0.162
I14	2.40	1.12	1.11	2.40	1.97	0.164
I15	2.20	1.10	1.10	2.20	1.84	0.167
I16	2.00	1.09	1.09	2.00	1.70	0.170
I17	1.80	1.08	1.07	1.80	1.57	0.174
I18	1.60	1.06	1.06	1.60	1.43	0.179
I19	1.40	1.04	1.04	1.40	1.29	0.184
I20	1.20	1.02	1.02	1.20	1.15	0.191
I21	1.00	1.00	1.00	1.00	1.00	0.200
I22	0.80	0.97	0.97	0.80	0.85	0.212
I23	0.60	0.94	0.94	0.60	0.69	0.228
I24	0.40	0.89	0.89	0.40	0.51	0.255
I25	0.20	0.82	0.80	0.20	0.31	0.309