

\$TITLE: Model M3-4a: TWOxTWOxONE Economy - Basics

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This model is a closed economy version of the classic Heckscher-Ohlin model: two goods and two factors, one consumer. Utility is treated as a produced good: quantity W, price PW.

Markets	Production Sectors			Consumers	
	X	Y	W	/	CONS
PX	/ 100		-100	/	
PY		/ 100	-100	/	
PW			200	/	-200
PL	/ -25	-75		/	100
PK	/ -75	-25		/	100

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PARAMETERS

```
LENDOW      labor endowment multiplier
KENDOW      capital endowment multiplier;
```

```
LENDOW = 1;
KENDOW = 1;
```

NONNEGATIVE VARIABLES

X activity level for X production
Y activity level for Y production
W activity level for the "production" of welfare from X Y

PX price of good X
PY price of good Y
PW price of a unit of welfare (real consumer-price index)
PL price of labor
PK price of capital

CONS income of the representative consumer;

EQUATIONS

PRF_X zero profit for sector X
PRF_Y zero profit for sector Y
PRF_W zero profit for sector W (Hicksian welfare index)

MKT_X supply-demand balance for commodity X
MKT_Y supply-demand balance for commodity Y
MKT_L supply-demand balance for primary factor L
MKT_K supply-demand balance for primary factor K
MKT_W supply-demand balance for aggregate demand

```
I_CONS income definition for CONS;  
  
*      Zero profit inequalities  
  
PRF_X..      100*(PL**0.25 * PK**0.75) =G= 100*PX;  
  
PRF_Y..      100*(PL**0.75 * PK**0.25) =G= 100*PY;  
  
PRF_W..      200*(PX**0.50 * PY**0.50) =G= 200*PW;  
  
*      Market clearance inequalities  
  
MKT_X..      100*X =G= 100 * W * PW / PX;  
  
MKT_Y..      100*Y =G= 100 * W * PW / PY;  
  
MKT_W..      200*W =E= CONS / PW;  
  
MKT_L..      100*LENDOW =G= 25 * X * PX / PL +  
                  75 * Y * PY / PL;  
  
MKT_K..      100*KENDOW =G= 75 * X * PX / PK +  
                  25 * Y * PY / PK;
```

* *Income balance equations (don't forget tax revenue)*

```
I_CONS..      CONS =E= 100*LENDOW*PL + 100*KENDOW*PK;
```

```
MODEL TWOxTWO /PRF_X.X, PRF_Y.Y, PRF_W.W,
           MKT_X.PX, MKT_Y.PY, MKT_L.PL,MKT_K.PK,
           MKT_W.PW,I_CONS.CONS /;
```

* *Choose a numeraire: real consumer price index*

```
PW.FX = 1;
```

* *Set initial values of variables:*

```
X.L=1; Y.L=1; W.L=1;
PX.L=1; PY.L=1; PK.L=1; PL.L=1;
CONS.L=200;
```

```
SOLVE TWOxTWO USING MCP;
```

* *counterfactual 1: double the endowment of labor*

```
LENDOW = 2;
```

```
SOLVE TWOxTWO USING MCP;
```

* *counterfactual 2: double both endowments from the benchmark*

```
LENDOW = 1;  
KENDOW = 2;  
SOLVE TWOxTWO USING MCP;
```

* counterfactual 3: double the endowment of both factors

```
LENDOW = 2;  
KENDOW = 2;  
SOLVE TWOxTWO USING MCP;
```

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We emphasize that the above formulation uses a simplifying trick: the marginal costs of X , Y , and W can be replaced by the producer prices P_X , P_Y , and P_W . This should "always" work because when marginal cost and price are not equal in equilibrium, the quantity is zero: marginal cost times quantity = producer price times quantity ($MC_X \cdot X = P_X \cdot X$ regardless of $X > 0$ or $X = 0$). But below is the model done "properly" with Shepard's lemma

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EQUATIONS

```
MKT_X2  
MKT_Y2  
MKT_L2  
MKT_K2
```

```
I_CONS2;

MKT_X2..      100*X =G= 100 * W * (PX**0.5 * PY**0.5) / PX;

MKT_Y2..      100*Y =G= 100 * W * (PX**0.5 * PY**0.5) / PY;

MKT_L2..      100*LENDOW =G= 25 * X * PL**0.25 * PK**0.75 / PL +
                  75 * Y * PL**0.75 * PK**0.25 / PL;

MKT_K2..      100*KENDOW =G= 75 * X * PL**0.25 * PK**0.75 / PK +
                  25 * Y * PL**0.75 * PK**0.25 / PK;

*           Income balance equations (don't forget tax revenue)

I_CONS2..      CONS =E= 100*LENDOW*PL + 100*KENDOW*PK;

MODEL TWOxTWOa /PRF_X.X, PRF_Y.Y, PRF_W.W,
               MKT_X2.PX, MKT_Y2.PY, MKT_L2.PL,MKT_K2.PK,
               MKT_W.PW,I_CONS2.CONS /;

LENDOW = 1; KENDOW = 1;

SOLVE TWOxTWOa USING MCP;

LENDOW = 2;
SOLVE TWOxTWOa USING MCP;
```

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Exercise: declare a parameter alpha, which is a productivity shift parameter

producing X. Higher alpha, more output per input.

Code this up. Hint: alpha will appear more than in the program.

Change alpha and interpret results.

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