```
$TITLE: Model M3-4b: Closed TWOxTWO Economy - adds taxes
* adds taxes to model M3-1

$ONTEXT
                  Production Sectors          Consumers
   Markets    |    X       Y        W     |       CONS
   ---------------------------------------------------
        PX    |  100              -100     |
        PY    |          100      -100     |
        PW    |                    200     |       -200
        PL    |  -25      -75              |        100
        PK    |  -75      -25              |        100
   ---------------------------------------------------
PX and PY will denote the consumer prices of X and Y
since PX(1+TX) = MC (marginal cost), the producer price of X (MC) is
PX/(1+TX) and similarly for Y

$OFFTEXT


PARAMETERS

 TX          ad-valorem tax rate for X sector inputs
 TY          ad-valorem tax rate for Y sector inputs
 LENDOW      labor endowment multiplier
 KENDOW      capital endowment multiplier;
```

```
TX = 0; TY = 0;
LENDOW = 1;
KENDOW = 1;

POSITIVE VARIABLES

 X       activity level for X production
 Y       activity level for Y production
 W       activity level for the "production" of welfare from X Y

 PX      price of good X
 PY      price of good Y
 PW      price of a unit of welfare (real consumer-price index)
 PL      price of labor
 PK      price of capital

 CONS    income of the representative consumer;

EQUATIONS

 PRF_X   zero profit for sector X
 PRF_Y   zero profit for sector Y
 PRF_W   zero profit for sector W (Hicksian welfare index)

 MKT_X   supply-demand balance for commodity X
```

```
 MKT_Y   supply-demand balance for commodity Y
 MKT_L   supply-demand balance for primary factor L
 MKT_K   supply-demand balance for primary factor L
 MKT_W   supply-demand balance for aggregate demand

 I_CONS  income definition for CONS;


*       Zero profit inequalities

PRF_X..      100*(PL**0.25 * PK**0.75) * (1+TX) =G= 100*PX;

PRF_Y..      100*(PL**0.75 * PK**0.25) * (1+TY) =G= 100*PY;

PRF_W..      200*(PX**0.50 * PY**0.50) =G= 200*PW;

*       Market clearance inequalities

MKT_X..      100*X =G= 100 * W * PW / PX;

MKT_Y..      100*Y =G= 100 * W * PW / PY;

MKT_W..      200*W =E= CONS / PW;

MKT_L..      100*LENDOW =G= 25 * X * (PX/(1+TX)) / PL +
                            75 * Y * (PY/(1+TY)) / PL;
```

```
MKT_K..       100*KENDOW =G= 75 * X * (PX/(1+TX)) / PK +
                             25 * Y * (PY/(1+TY)) / PK;

*       Income balance equations (don't forget tax revenue)

I_CONS..      CONS =E= 100*LENDOW*PL + 100*KENDOW*PK +
                       TX*100*X*(PX/(1+TX)) +
                       TY*100*Y*(PY/(1+TY));

MODEL TWOxTWO /PRF_X.X, PRF_Y.Y, PRF_W.W,
                  MKT_X.PX, MKT_Y.PY, MKT_L.PL,MKT_K.PK,
                  MKT_W.PW,I_CONS.CONS /;


*       Chose a numeraire: real consumer price index

PW.FX = 1;

*       Set initial values of variables:

X.L=1; Y.L=1; W.L=1; PX.L=1; PY.L=1; PK.L=1; PL.L=1;
CONS.L=200;

SOLVE TWOxTWO USING MCP;
```

```
*   counterfactual 1: 50% tax on X

TX = 0.5;
SOLVE TWOxTWO USING MCP;

*   counterfactual 1: 50% tax on X and Y

TX = 0.5;
TY = 0.5;
SOLVE TWOxTWO USING MCP;

*   counterfactual 2: zero taxes, double the labor endowment

TX = 0;
TY = 0;
LENDOW = 2;
SOLVE TWOxTWO USING MCP;

*   counterfactual 3: double both endowments from the benchmark

LENDOW = 2;
KENDOW = 2;
SOLVE TWOxTWO USING MCP;
```

```
$ontext
We emphasize that the above formulation uses a simplifying trick:
the marginal costs of X, Y, and W can be replaced by the producer
prices PX/(1+TX), PY/(1+TY), and PW.  This should "always" work
because when marginal cost and price are not equal in equilbrium,
the quantity is zero: marginal cost times quantity = producer
price times quantity (MC_X*X = PX*X regardless of X > 0 or X = 0).
But below is the model done "properly" with Shepard's lemma
$offtext


EQUATIONS
 MKT_X2
 MKT_Y2
 MKT_L2
 MKT_K2
 I_CONS2;

MKT_X2..      100*X =G= 100 * W * (PX**0.5 * PY**0.5) / PX;

MKT_Y2..      100*Y =G= 100 * W * (PX**0.5 * PY**0.5) / PY;

MKT_L2..      100*LENDOW =G= 25 * X * PL**0.25 * PK**0.75 / PL +
                             75 * Y * PL**0.75 * PK**0.25 / PL;
```

```
MKT_K2..      100*KENDOW =G= 75 * X * PL**0.25 * PK**0.75 / PK +
                             25 * Y * PL**0.75 * PK**0.25 / PK;

*       Income balance equations (don't forget tax revenue)

I_CONS2..     CONS =E= 100*LENDOW*PL + 100*KENDOW*PK +
                       TX*100*X*(PL**0.25*PK**0.75) +
                       TY*100*Y*(PL**0.75*PK**0.25);

MODEL TWOxTWOa /PRF_X.X, PRF_Y.Y, PRF_W.W,
                MKT_X2.PX, MKT_Y2.PY, MKT_L2.PL,MKT_K2.PK,
                MKT_W.PW,I_CONS2.CONS /;

TX = 0; TY = 0;
LENDOW = 1; KENDOW = 1;

SOLVE TWOxTWOa USING MCP;

TX = 0.5;
SOLVE TWOxTWOa USING MCP;


$ontext
Exercise: declare a parameter alpha, which is a productivity shift
parameter producting X.  Higher alpha, more output per input.
```

```
*Code this up.  Hint: alpha will appear more than in the program.
*Change alpha and interpret results.
$offtext
```