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$TITLE: M5-4.GMS  Balance a matrix (micro-consistent data) as an NLP
* Simple micro-consistency example: least squares to balance matrix
* Minimize sum of squared errors in adjusting data to get zero row and
* column sums

SETS R /PX, PY, PL/
     C /X, Y, CONS/;

PARAMETERS
 Z0(R,C)       original unbalanced data
 BENCHC(R,C)  new balanced matrix
 BENCH3(R,C)  further constraints - no intermediates or joint outputs;

TABLE BENCH(*,*)
      X      Y    CONS
PX    10      0   -11
PY     0     11   -12
PL    -8    -12    21 ;

Z0(R,C) = BENCH(R,C);

DISPLAY Z0;

VARIABLES
 Z(R,C)  new adjusted data
 DEV     sum of squared errors from original data;
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EQUATIONS
 SUMSQ         objective function: sse
 ROWSUM(R)     sum of elements in row R
 COLSUM(C)     sum of elements in column C;

*SUMSQ..       DEV =E= SUM(R, SUM(C,(Z(R,C) - Z0(R,C))**2));

SUMSQ..       DEV =E= SUM(R, SUM(C,(Z(R,C)-Z0(R,C))*(Z(R,C)-Z0(R,C))));

ROWSUM(R).. SUM(C, Z(R,C)) =E= 0.;

COLSUM(C).. SUM(R, Z(R,C)) =E= 0.;

MODEL MCONS /ALL/;


DEV.L = 20;

* set some starting values
Z.L(R,C) = Z0(R,C) + 1;

SOLVE MCONS USING NLP MINIMIZING DEV;

BENCHC(R,C) = Z.L(R,C);
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DISPLAY BENCH, BENCHC;


*suppose that we with to rule out intermediate inputs and joint outputs

EQUATIONS
 CONST1
 CONST2;

CONST1..    Z("PY","X") =E= 0;
CONST2..    Z("PX","Y") =E= 0;

MODEL MCONS2 /ALL/;

DEV.L = 20;

Z.L(R,C) = Z0(R,C) + 1;

SOLVE MCONS2 USING NLP MINIMIZING DEV;

BENCH3(R,C) = Z.L(R,C);

DISPLAY BENCH, BENCHC, BENCH3;
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