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$TITLE  M6-4.GMS: Economy with two consumers, public good,
*        optimal provision with an endogenous tax rate, Samuelson rule

$ONTEXT
 Samuelson rule for optimal provision, PG = PG1 + PG2
  introduces an auxiliary variable and constraint equation
 Here is the tax rate is a VARIABLE, set optimally
 Generalizes M6-3.gms: two consumers with different preferences
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| | Production Sectors | | | | | Consumers | | |
Markets|	X	Y	G	W1	W2	CONS1	CONS2	GOVT
PX |	100			-50	-50			
PY |		100		-50	-50			
PG |			50					-50
PL |	-80	-80	-40			100	100	
TAX |	-20	-20	-10					50
PW1 |				125		-125		
PW2 |					125		-125	
PG1 |				-25		25		
PG2 |					-25		25	

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$OFFTEXT
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PARAMETERS
 SHX1, SHY1, SHG1   shares of X Y and G in consumer 1's utility
 SHX2, SHY2, SHG2   shares of X Y and G in consumer 2's utility;

SHG1 = 0.2;
SHX1 = 0.5 - SHG1/2;
SHY1 = 0.5 - SHG1/2;
SHG2 = 0.2;
SHX2 = 0.5 - SHG2/2;
SHY2 = 0.5 - SHG2/2;

POSITIVE VARIABLES
   X   Activity level for sector X,
   Y   Activity level for sector Y,
   W1   Activity level for sector W1,
   W2   Activity level for sector W2,
   G   Activity level for government sector,

   PX   Price index for commodity X,
   PY   Price index for commodity Y,
   PL   Price index for primary factor L,
   PW1   Price index for welfare 1(expenditure function),
   PW2   Price index for welfare 2(expenditure function),
   PG1   Private valuation of the public good (consumer 1),
   PG2   Private valuation of the public good (consumer 2),

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    PG        Price (marginal cost) of the public good


    GOVT      Budget restriction for government,
    CONS1     Income definition for CONS1,
    CONS2     Income definition for CONS2,


    LGP       Level of government provision
    TAX       Uniform value-added tax rate;

EQUATIONS
    PRF_X     Zero profit for sector X
    PRF_Y     Zero profit for sector Y
    PRF_W1    Zero profit for sector W1
    PRF_W2    Zero profit for sector W2
    PRF_G     Zero profit in government sector


    MKT_X     Supply-demand balance for commodity X
    MKT_Y     Supply-demand balance for commodity Y
    MKT_L     Supply-demand balance for primary factor L
    MKT_W1    Supply-demand balance for consumer 1
    MKT_W2    Supply-demand balance for consumer 2
    MKT_G1    Private valuation of the public good (consumer 1)
    MKT_G2    Private valuation of the public good (consumer 2)
    MKT_G     Supply-demand balance for commodity G


    I_G       Budget restriction for government
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   I_CONS1 Income definition for CONS1
   I_CONS2 Income definition for CONS2

   A_LGP   Auxiliary for government provision
   A_TAX   Auxiliary for government provision;
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*   *Zero profit conditions:*

```
PRF_X..  80*PL * (1+TAX) =G= 100*PX;


PRF_Y..  80*PL * (1+TAX) =G= 100*PY;


PRF_W1.. 125*PX**(SHX1) * PY**(SHY1) * (PG1/0.5)**(SHG1)
          =E= 125*PW1;


PRF_W2.. 125*PX**(SHX2) * PY**(SHY2) * (PG2/0.5)**(SHG2)
          =E= 125*PW2;


PRF_G..  40*PL * (1+TAX) =G= 50*PG;
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*   *Market clearing conditions:*

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MKT_X..  100*X =G= 125*SHX1*W1*PW1/PX + 125*SHX2*W2*PW2/PX ;


MKT_Y..  100*Y =G= 125*SHY1*W1*PW1/PY + 125*SHY2*W2*PW2/PY;
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MKT_W1.. 125*W1 =G= CONS1/PW1;

MKT_W2.. 125*W2 =G= CONS2/PW2;

MKT_L..  200 =G= (80*X + 80*Y + 40*G);

MKT_G1.. 50 * LGP =G= 125*SHG1 * W1 * PW1/PG1;

MKT_G2.. 50 * LGP =G= 125*SHG2 * W2 * PW2/PG2;

MKT_G..  50*G =G= GOVT/ PG;

*        Income constraints:

I_G..       GOVT =G= PL*(80*X + 80*Y + 40*G )*TAX;

I_CONS1.. CONS1 =E= 100*PL + 50*LGP*PG1;

I_CONS2.. CONS2 =E= 100*PL + 50*LGP*PG2;

*        Auxiliary constraints:

A_LGP..  LGP =E= G;

A_TAX..  PG =E= PG1 + PG2;
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MODEL PUBGOOD2 /PRF_X.X, PRF_Y.Y, PRF_W1.W1, PRF_W2.W2,
                PRF_G.G,
                MKT_X.PX, MKT_Y.PY, MKT_L.PL,
                MKT_W1.PW1, MKT_W2.PW2,
                MKT_G.PG, MKT_G1.PG1, MKT_G2.PG2,
                I_G.GOVT, I_CONS1.CONS1, I_CONS2.CONS2,
                A_LGP.LGP, A_TAX.TAX /;


X.L      =1;
Y.L      =1;
W1.L     =1;
W2.L     =1;
G.L      =1;


PL.FX    =1;
PX.L     =1;
PY.L     =1;


PG.L     =1;
PW1.L    =1;
PW2.L    =1;
PG1.L   =0.5;
PG2.L   =0.5;


CONS1.L =125;
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CONS2.L =125;
GOVT.L  =50;

LGP.L   =1;

TAX.L   =0.25;

PUBGOOD2.ITERLIM = 0;
SOLVE PUBGOOD2 USING MCP;
PUBGOOD2.ITERLIM = 2000;
SOLVE PUBGOOD2 USING MCP;

*   Change consumer 1's preferences, higher preference for the
*   public good, which now has a Cobb-Douglas share of 0.3

SHG1 = 0.3;
SHX1 = 0.5 - SHG1/2;
SHY1 = 0.5 - SHG1/2;

*PUBGOOD2.ITERLIM = 0;
SOLVE PUBGOOD2 USING MCP;



TAX.FX = 0.25;

SOLVE PUBGOOD2 USING MCP;
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