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$TITLE  M6-5.GMS: Public intermediate good with optimal provision
* technique for modeling infrastructure for example

$ONTEXT
             Production Sectors          Consumers

Markets|    X    Y    G   W1         CONS1     GOVT
-------------------------------------------------------------
  PX   |  100             -100
  PY   |       100        -100
  PG   |             50                        -50
  PL   |  -80  -80  -40               200
  TAX  |  -20  -20  -10                         50
  PW   |                   200        -200
-------------------------------------------------------------
  X = ALPHA*L  ALPHA = F(G)  ALPHA viewed as exogenous by firms

$OFFTEXT

PARAMETERS
 SHX, SHY  shares of X and Y in consumer's utility
 INFPROD   productivity parameter of the public good in X output
 WELF;

SHX = 0.5;
SHY = 0.5;
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INFPROD = 0;

POSITIVE VARIABLES
   X       Activity level for sector X
   Y       Activity level for sector Y
   W       Activity level for sector W
   G       Activity level for government sector

   PX      Price index for commodity X
   PY      Price index for commodity Y
   PG      Private valuation of the public good
   PL      Price index for primary factor L
   PW      Price index for welfare 1(expenditure function)

   GOVT    Budget restriction for government
   CONS    Income definition for CONS

   TAX     Uniform value-added tax rate
   ALPHA   Public intermediary good multiplier on productivity;

EQUATIONS
   PRF_X   Zero profit for sector X
   PRF_Y   Zero profit for sector Y
   PRF_W   Zero profit for sector W1
   PRF_G   Zero profit in government sector
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   MKT_X   Supply-demand balance for commodity X
   MKT_Y   Supply-demand balance for commodity Y
   MKT_G   Supply-demand balance for commodity G
   MKT_L   Supply-demand balance for primary factor L
   MKT_W   Supply-demand balance for consumer 1

   I_G     Budget restriction for government
   I_CONS  Income definition for CONS

   A_TAX   Auxiliary for government provision
   INFRA   Auxiliary for public intermediate good calculation;

*       Zero profit conditions:

PRF_X..  80*PL * (1+TAX)/ALPHA =G= 100*PX;

PRF_Y..  80*PL * (1+TAX) =G= 100*PY;

PRF_W .. 200*PX**(SHX) * PY**(SHY) =E= 200*PW;

PRF_G..  40*PL * (1+TAX) =G= 100*PG;

*       Market clearing conditions:

MKT_X..  100*X =G= 200*SHX*W*PW/PX;
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MKT_Y..  100*Y =G= 200*SHY*W*PW/PY;

MKT_G..  100*G =G= GOVT/ PG;

MKT_L..  200 =G= (80*X/ALPHA + 80*Y + 40*G);

MKT_W..  200*W =G= CONS/PW;

*       Income constraints:

I_G..    GOVT =G= PL*(80*X/ALPHA + 80*Y + 40*G )*TAX;

I_CONS.. CONS =E= 200*PL;

*       Auxiliary constraints:

A_TAX..  PG =E= PX*INFPROD*(X/ALPHA);

INFRA..  ALPHA =E= 1 + INFPROD*G;


MODEL PUBINT    /PRF_X.X, PRF_Y.Y, PRF_W.W, PRF_G.G,
                 MKT_X.PX, MKT_Y.PY, MKT_L.PL, MKT_W.PW, MKT_G.PG,
                 I_G.GOVT, I_CONS.CONS,
                 A_TAX.TAX, INFRA.ALPHA /;
```

```
X.L     =1;
Y.L     =1;
W.L     =1;
G.L     =1;
PL.FX   =1;
PX.L    =1;
PY.L    =1;
PG.L    =0.5;
PW.L    =1;
CONS.L  =200;
GOVT.L  =50;
ALPHA.L = 1;
TAX.L   = .25;

PUBINT.ITERLIM = 0;
SOLVE PUBINT USING MCP;

* with INFPROD = 0 initially, the optimal tax should be zero

PUBINT.ITERLIM = 2000;
SOLVE PUBINT USING MCP;

* now set INFPROD = 2, optimal tax and provision should be positive

INFPROD = 2;
TAX.L = 0.25; G.L = 1;
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```
SOLVE PUBINT USING MCP;

WELF = W.L*100;
DISPLAY WELF;


* now let's check by "brute force" whether the answer is right
* loop over fixed values of TAX

SETS I /I1*I15/;

PARAMETERS
 WELFARE(I)
 TAXRATE(I);

LOOP(I,
TAX.FX = 0.29 + 0.01*ORD(I);

SOLVE PUBINT USING MCP;

WELFARE(I) = 100*W.L;
TAXRATE(I) = TAX.L;

);

DISPLAY TAXRATE, WELFARE;
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