

\$TITLE M6-6c.GMS: Pollution tax set optimally via a
 * "first-order condition"
 * TX is set by an equation equation the price of X to it's full cost:
 * $PX = PL + PCA$

\$ONTEXT

*This model is a closed economy: two goods and one factor, one consumer
 Pollution is generated by the production of X, pollution reduces utility
 Pollution is modeled as a reduction in the endowment of CLEAN AIR
 Initial endowment of clear air is 200, with 100 reduced by X pollution
 and 100 entering utility.*

Markets	Production Sectors			Consumers
	X	Y	W	
PX	100		-100	
PY		100	-100	
PW			300	-300
PL	-100	-100		200
PCA			-100	(200 - 100)

\$OFFTEXT

PARAMETERS

POLINT pollution intensity multiplier
WELOPT welfare under the optimal tax
TAXOPT value of the optimal tax;

POLINT = 1;

NONNEGATIVE VARIABLES

X activity level for X production
Y activity level for Y production
W activity level for the "production" of welfare from X Y

PX price of good X
PY price of good Y
PCA price of clean air
PW price of a unit of welfare (real consumer-price index)
PL price of labor

CONS income of the representative consumer
POL pollution
TX pollution tax;

EQUATIONS

PRF_X zero profit for sector X
 PRF_Y zero profit for sector Y
 PRF_W zero profit for sector W (Hicksian welfare index)

 MKT_X supply-demand balance for commodity X
 MKT_Y supply-demand balance for commodity Y
 MKT_CA market for clean air (determines shadow value PCA)
 MKT_L supply-demand balance for primary factor L
 MKT_W supply-demand balance for aggregate demand

 I_CONS income definition for CONS
 PPOL pollution caused by production - consumption of X
 ATX sets pollution tax optimally;

* *Zero profit inequalities*

PRF_X.. $100 * PL * (1 + TX) = G = 100 * PX;$

PRF_Y.. $100 * PL = G = 100 * PY;$

PRF_W.. $200 * (PX^{**}(1/3) * PY^{**}(1/3) * PCA^{**}(1/3)) = G = 200 * PW;$

* *Market clearance inequalities*

MKT_X.. 100*X =G= 100 * W * PW / PX;

MKT_Y.. 100*Y =G= 100 * W * PW / PY;

MKT_CA.. 200-100*POL =G= 100 * W * PW / PCA;

MKT_W.. 300*W =E= CONS / PW;

MKT_L.. 200 =G= 100*X + 100*Y;

* *Income balance equations (don't forget tax revenue)*

I_CONS.. CONS =E= 200*PL + (200-100*POL)*PCA + TX*100*X*PL;

PPOL.. 100*POL =G= POLINT*100*X;

ATX.. PX =E= PL + PCA*POLINT;

* *or since $PX = PL*(1 + TX)$, equivalently*

*ATX.. TX =E= PCA*POLINT / PL;

MODEL ALGEBRAIC /PRF_X.X, PRF_Y.Y, PRF_W.W,
 MKT_X.PX, MKT_Y.PY, MKT_CA.PCA, MKT_L.PL,
 MKT_W.PW, I_CONS.CONS, PPOL.POL, ATX.TX /;

* *Chose a numeraire: real consumer price index*

PW.FX = 1;

* *Set initial values of variables:*

X.L=1; Y.L=1; W.L=1; PX.L=1; PY.L=1; PL.L=1; POL.L = 1; PCA.L = 1;
CONS.L=300;

ALGEBRAIC.ITERLIM = 0;
SOLVE ALGEBRAIC USING MCP;
ALGEBRAIC.ITERLIM = 1000;
SOLVE ALGEBRAIC USING MCP;

WELOPT = 100*W.L;
TAXOPT = TX.L;
DISPLAY WELOPT, TAXOPT;

POLINT = 1.5;

SOLVE ALGEBRAIC USING MCP;

WELOPT = 100*W.L;
TAXOPT = TX.L;
DISPLAY WELOPT, TAXOPT;