

Chapter 2

Extensions of the Simple Model

In this chapter, we will introduce additional features that are commonly encountered and used in applied situations. Each feature is introduced one at a time to the basic two-good closed-economy model of the previous section. Obviously, models of real economies will require the modeler to use many of these features simultaneously, but considering them one by one may be a quicker avenue to understanding each building block than attempting to understand complex, multi-sector models all at one go.

All the models in this chapter are two-good, closed-economy models. To make each feature as transparent as possible, no taxes are present in the benchmark data. Taxes in the initial benchmark data and how to model those are introduced in chapter 3. Here are the model numbers, with a short description of the features added. “M2X” denotes Chapter 2, model number X.

M21.GMS	two goods, two factors, one household (same as M1_MPS.GMS)
M22.GMS	introduces intermediate inputs and nesting
M23.GMS	introduces joint production
M24.GMS	introduces the use of specific factors
M25.GMS	use of an initially slack activity (e.g., modeling tax avoidance)
M26.GMS	introduces a labor supply or labor/leisure choice activity
M27.GMS	two forms of labor supply, such as to formal/informal sectors
M28.GMS	two households with different preferences and endowments
M29.GMS	introduces Stone-Geary (LES) preferences to model non-homothetic demand

We will not go through the first model again, but simply include it for completeness in this chapter under a name consistent with the full set of models. You could use it to try some experiments.

Exercises:

- (1) explain the factor-price changes in the two counterfactuals relative to the benchmark.
- (2) change the capital endowment to 200, along with a labor endowment of 200. You should show that all quantities double from the base case, and all prices are unchanged.
- (3) Starting with the benchmark, lower the tax on X inputs to 0.25 and put a tax of 0.25 on Y inputs. What happens to welfare?
- (4) Starting with the benchmark, assume that only 50% of the tax revenue is redistributed back to the consumer.
- (5) change the X production functions to fixed coefficients. That is,

$$X = \min[L_x, K_x]; \quad \text{Hint: the cost function} \quad c_x = 0.25 * p_l + 0.75 * p_k$$

(A) reproduce the benchmark, (B) Run the counterfactuals and see what difference this makes.

Model M22 introduces intermediate inputs and nesting

We begin with model M22, which introduces intermediate goods, and a simple CES nesting notation that permits the modeler to specify different elasticities of substitution between different groups of inputs.

This model is shown below. X and Y sectors each use the other sector's output as an input, so that each sector has three inputs. With three inputs, there may be different elasticities of substitution between different pairs of these inputs. Consider the production block for the Y sector.

```
$PROD:Y s:0.75 va:1
      O:PY Q:120
      I:PX Q: 20
      I:PL Q: 60 va:
      I:PK Q: 40 va:
```

This specifies a two-level nesting structure. Labor and capital are combined in the both nest with an elasticity of substitution of 1 (va:1). We use VA to denote “value added” although this is not required and just “a” could be used. The important thing is the syntax used to specify the structure. Think of the inputs denoted with VA as combined to produce a composite input, in this case termed value added. The top level of the CES function specifies an elasticity of substitution of 0.75 (s:0.75) between the intermediate input, in this case X, and the value-added composite of capital and labor. In some models this is specified as equal to zero, indicating that a fixed amount of the intermediate input is required for each unit of output. A similar structure is specified for the X sector, and in addition we all for taxes on X sector inputs.

Following the MPS/GE version of the model, we present the code for the MCP version with all equations written out. This is done for two purposes. First, it again helps you to understand exactly what MPS/GE is doing with the simple the specification that you have given to it. Second, it should help convince you of the advantages of MPS/GE. Finding and correctly specifying the correct CES functions for the X and Y sectors is not an easy manner. But you are not finished there. You also have to correctly apply Shepard's lemma to find factor demands for use in the factor-market-clearing equations. While it is very important to understand exactly what MPS/GE is doing in the background, we hope this will also convince you to let this higher-level software do the dirty work.

\$TITLE Model M22: Closed Economy 2X2 with Intermediate Inputs and Nesting

\$ONTEXT

Markets	Production Sectors			Consumers
	X	Y	W	CONS
PX	120	-20	-100	
PY	-20	120	-100	
PW			200	-200
PL	-40	-60		100
PK	-60	-40		100

\$OFFTEXT

PARAMETERS

TX;

TX = 0;

\$ONTEXT

\$MODEL: M22

\$SECTORS:

X ! Activity level for sector X
 Y ! Activity level for sector Y
 W ! Activity level for sector W (Hicksian welfare index)

\$COMMODITIES:

PX ! Price index for commodity X
 PY ! Price index for commodity Y
 PL ! Price index for primary factor L
 PK ! Price index for primary factor K
 PW ! Price index for welfare (expenditure function)

\$CONSUMERS:

CONS ! Income level for consumer CONS

\$PROD:X s:0.5 va:1

O:PX Q:120

I:PY Q: 20

I:PL Q: 40 va: A:CONS T:TX

I:PK Q: 60 va: A:CONS T:TX

```

$PROD:Y s:0.75 va:1
O:PY Q:120
I:PX Q: 20
I:PL Q: 60 va:
I:PK Q: 40 va:

```

```

$PROD:W s:1
O:PW Q:200
I:PX Q:100
I:PY Q:100

```

```

$DEMAND:CONS
D:PW Q:200
E:PL Q:100
E:PK Q:100

```

```

$OFFTEXT
$SYSINCLUDE mpsgeset M22

```

```
PW.FX = 1;
```

```

$INCLUDE M22.GEN
SOLVE M22 USING MCP;

```

```
* Counterfactual: 100% tax on X sector inputs:
```

```

TX = 1.0;
$INCLUDE M22.GEN
SOLVE M22 USING MCP;

```

```
* Algebraic representation -- note the complexity of two-level
* CES functions which are automatically generated within MPSGE.
```

EQUATIONS

```

PRF_X Zero profit for sector X
PRF_Y Zero profit for sector Y
PRF_W Zero profit for sector W (Hicksian welfare index)

MKT_X Supply-demand balance for commodity X
MKT_Y Supply-demand balance for commodity Y
MKT_L Supply-demand balance for primary factor L
MKT_K Supply-demand balance for primary factor L
MKT_W Supply-demand balance for aggregate demand

I_CONS Income definition for CONS;

```

```

PRF_X.. 120 * ( 1/6 * PY**(1-0.5) + 5/6 * (PL**0.4 * PK**0.6 *
          (1+TX))**(1-0.5))**(1/(1-0.5)) =E= 120 * PX;

PRF_Y.. 120 * ( 1/6 * PX**(1-0.75) + 5/6 * (PL**0.6 * PK**0.4)**
          (1-0.75) )**(1/(1-0.75)) =E= 120 * PY;

PRF_W.. 200 * PX**0.5 * PY**0.5 =E= 200 * PW;

MKT_X.. 120 * X =E= 100 * W * PX**0.5 * PY**0.5 / PX +
          20*Y*(PY/PX)**0.75;

MKT_Y.. 120 * Y =E= 100 * W * PX**0.5 * PY**0.5 / PY +
          20*X*(PX/PY)**0.5;

MKT_W.. 200 * W =E= CONS / PW;

MKT_L.. 100 =E= 40 * X * (PX/((1+TX)*PL**0.4*PK**0.6))**0.5
          * PL**0.4 * PK**0.6 / PL +
          60 * Y * (PY/(PL**0.6 * PK**0.4))**0.75
          * PL**0.6 * PK**0.4 / PL;

MKT_K.. 100 =E= 60 * X * (PX/((1+TX)*PL**0.4*PK**0.6))**0.5
          * PL**0.4 * PK**0.6 / PK +
          40 * Y * (PY/(PL**0.6 * PK**0.4))**0.75
          * PL**0.6 * PK**0.4 / PK;

I_CONS.. CONS =E= 100*PL + 100*PK + TX * 100 * X * PL**0.4*PK**0.6 *
          (PX/((1+TX)*PL**0.4*PK**0.6))**0.5;

MODEL ALGEBRAIC /PRF_X.X, PRF_Y.Y, PRF_W.W, MKT_X.PX, MKT_Y.PY,
MKT_L.PL, MKT_K.PK, MKT_W.PW, I_CONS.CONST /;

*      Check the benchmark:

X.L=1; Y.L=1; W.L=1; PX.L=1; PY.L=1; PK.L=1; PW.L=1; CONS.L=200;

TX = 0;
SOLVE ALGEBRAIC USING MCP;

*      Solve the same counterfactual:

TX = 1;
SOLVE ALGEBRAIC USING MCP;

```

Excercises:

- (1) Revise the X sector production block so that the elasticity of substitution between Y and value added is 0: there are fixed coefficients between Y and value added.

```
$PROD:X s:0 va:1
      O:PX  Q:120
      I:PY  Q: 20
      I:PL  Q:40 va:      A:CONS  T:TX
      I:PK  Q:60 va:      A:CONS  T:TX
```

run the model and verify that the benchmark is reproduced. See if you can interpret the differences between the new counterfactual results and the initial results.

- (2) Rewrite the algebraic model in accordance with the new value of the upper level nest, and verify that you obtain identical solution values.

Model 23 introduces joint production

This model continues the same theme, by noting that a given sector can have several output as well as inputs. Here we assume that two sectors, A and B that both produce final outputs X and Y, but sector A is relatively specialized in producing good X and sector B is more specialized in producing good Y. MPS/GE allows the modeler to specify a constant elasticity of transformation (CET) between pairs of outputs, and a nesting structure can also be used in more complicated cases. Here is the production block for B. The transformation elasticity between the two goods is equal to 1.5 (t:1.5) and the substitution elasticity between inputs is 1 (s:1).

```
$PROD:B t:1.5 s:1
      O:PX  Q:20
      O:PY  Q:80
      I:PL  Q:60
      I:PK  Q:40
```

Now the model itself, followed by the MCP version.

\$TITLE Model M23: Closed Economy 2x2 with Joint Production

\$ONTEXT

Markets	Production Sectors			Consumers
	A	B	W	CONS
PX	80	20	-100	
PY	20	80	-100	
PW			200	-200
PL	-40	-60		100
PK	-60	-40		100

\$OFFTEXT

PARAMETERS

TA;

TA = 0;

\$ONTEXT

\$MODEL: M23

\$SECTORS:

A ! Activity level for sector A (80:20 for X:Y)
 B ! Activity level for sector B (20:80 for X:Y)
 W ! Activity level for sector W (Hicksian welfare index)

\$COMMODITIES:

PX ! Price index for commodity X
 PY ! Price index for commodity Y
 PL ! Price index for primary factor L
 PK ! Price index for primary factor K
 PW ! Price index for welfare (expenditure function)

\$CONSUMERS:

CONS ! Income level for consumer CONS

\$PROD:A t:2.0 s:1

O:PX Q:80

O:PY Q:20

I:PL Q:40 A:CONS T:TA

I:PK Q:60 A:CONS T:TA


```

$PROD:B t:1.5 s:1
        O:PX Q:20
        O:PY Q:80
        I:PL Q:60
        I:PK Q:40

```

```

$PROD:W s:1
        O:PW Q:200
        I:PX Q:100
        I:PY Q:100

```

```

$DEMAND:CONS
        D:PW Q:200
        E:PL Q:100
        E:PK Q:100

```

```

$OFFTEXT
$SYSINCLUDE mpsgeset M23

```

```

        PW.FX = 1;

```

```

$INCLUDE M23.GEN
SOLVE M23 USING MCP;

```

```

*          Counterfactual: 10% tax on X sector inputs:

```

```

TA = 0.10;
$INCLUDE M23.GEN
SOLVE M23 USING MCP;

```

```

*          Counterfactual: 100% tax on X sector inputs:

```

```

TA = 1.00;
$INCLUDE M23.GEN
SOLVE M23 USING MCP;

```

```

* now the mcp version, which again shows you the simplifying features
* of MPS/GE

```

```

EQUATIONS

```

```

        PRF_A   Zero profit for sector X
        PRF_B   Zero profit for sector Y
        PRF_W   Zero profit for sector W (Hicksian welfare index)

```

```

        MKT_X   Supply-demand balance for commodity X
        MKT_Y   Supply-demand balance for commodity Y

```

MKT_L Supply-demand balance for primary factor L
MKT_K Supply-demand balance for primary factor
MKT_W Supply-demand balance for aggregate demand

I_CONS Income definition for CONS;

* Write the profit constraints as inequalities -- the tax
* can cause sector A to shut down completely:

PRF_A.. 100 * PL**0.4 * PK**0.6 * (1+TA) =G=
100 * (0.8 * PX**(1+2.0) + 0.2 * PY**(1+2.0))**(1/(1+2.0));

PRF_B.. 100 * PL**0.6 * PK**0.4 =G=
100 * (0.2 * PX**(1+1.5) + 0.8 * PY**(1+1.5))**(1/(1+1.5));

PRF_W.. 200 * PX**0.5 * PY**0.5 =E= 200 * PW;

MKT_X.. 80 * A * (PX/(0.8*PX**(1+2.0)+0.2*PY**(1+2.0))**(1/(1+2.0)))**2
+ 20 * B * (PX/(0.2*PX**(1+1.5)+0.8*PY**(1+1.5))**(1/(1+1.5)))**1.5
=E= 100 * W * PX**0.5 * PY**0.5 / PX;

MKT_Y.. 20 * A * (PY/(0.8*PX**(1+2.0)+0.2*PY**(1+2.0))**(1/(1+2.0)))**2.0
+ 80 * B * (PY/(0.2*PX**(1+1.5)+0.8*PY**(1+1.5))**(1/(1+1.5)))**1.5
=E= 100 * W * PX**0.5 * PY**0.5 / PY;

MKT_W.. 200 * W =E= CONS / PW;

MKT_L.. 100 =E= 40 * A * PL**0.4 * PK**0.6 / PL +
60 * B * PL**0.6 * PK**0.4 / PL;

MKT_K.. 100 =E= 60 * A * PL**0.4 * PK**0.6 / PK +
40 * B * PL**0.6 * PK**0.4 / PK;

I_CONS.. CONS =E= 100*PL + 100*PK + TA*100*A*PL**0.4*PK**0.6;

MODEL ALGEBRAIC /PRF_A.A, PRF_B.B, PRF_W.W, MKT_X.PX, MKT_Y.PY,
MKT_L.PL, MKT_K.PK, MKT_W.PW, I_CONS.CONST /;

* Check the benchmark:

A.L=1; B.L=1; W.L=1; PX.L=1; PY.L=1; PK.L=1; PW.L=1; CONS.L=200;

TA = 0;

SOLVE ALGEBRAIC USING MCP;

* Solve the same counterfactuals:

TA = 0.10;

SOLVE ALGEBRAIC USING MCP;

TA = 1.00;

SOLVE ALGEBRAIC USING MCP;

Exercises:

- (1) Try a higher elasticity of transformation between output in the two sectors, such as $t = 10$. Try to guess ahead of time as to how this might affect the equilibrium outputs of the two sectors following imposition of the tax. Recall that the A sector is the sector which is relatively good at producing X.
- (3) Assume that the elasticity of transformation between outputs in each sector is zero: that is, outputs come in fixed proportions. Run the mpsge model. Now see if you can modify the algebraic model with this new assumption.
- (2) While it is harder to find a good discussion of CET functions in a textbook (which generally concentrate on the CES input side), see if you can verify that the MCP version is correct for one of the sectors. Let σ be the elasticity of transformation. CET functions have the form:

$$(aX^\alpha + bY^\alpha)^{1/\alpha} = F(L, K) \quad \text{where } \alpha \geq 1 \quad \sigma = \frac{1}{\alpha - 1}$$

Model 24 introduces the use of specific factors

There are some inherent difficulties in Heckscher-Ohlin type models, in which all factors are used in and are mobile between sectors. In particular, trade theory demonstrates that when there are more goods than factors, there are “flats” in the aggregate transformation surface of the economy, and open economies tend to specialize in only as many goods as factors. This is, of course, inconsistent with real data, which generally show economies producing something in all industries.

There are several ways around this problem. One is the “Armington” assumption introduced in the next chapter. A second is to assume a portion of capital, or other factors (e.g., resources, land) is fixed or sector specific in each sector. That is what we do in this example. We take the data matrix of M21 (M1_MPS) and split capital into three factors: (i) capital that is mobile between the two sectors, (ii) capital specific to Y, and (iii) capital specific to X.

This assumption helps “convexify” the model and allow positive production of all goods.

It is also a useful assumption when data is too aggregated, such as modeling the valued added in an energy sector as comprised of just “capital” and labor. Designating a portion of that capital as sector-specific resources, allows for more realistic policy experiments. Second, the returns to the specific factor have a nice interpretation as resource rents.

Here is the data matrix of model M21

Markets	Production Sectors			Consumers
	X	Y	W	CONS
PX	100		-100	
PY		100	-100	
PW			200	-200
PL	-25	-75		100
PK	-75	-25		100

Now designate part of the capital in each sector as fixed in that sector, creating a four-factor model.

Markets	Production Sectors			Consumers
	X	Y	W	CONS
PX	100		-100	
PY		100	-100	
PW			200	-200
PL	-25	-75		100
PK	-50	-15		65
PKX	-25			25
PKY		-10		10

\$TITLE Model M24: Closed Economy 2x2 with Specific Factors

\$ONTEXT

Here is the initial data matrix for example M21 (also M1_MPS). As noted in the text description, it is technically useful to interpret a portion of capital in each sector as sector specific. Or it can in fact be a separate factor such as land or resources.

Markets	Production Sectors			Consumers
	X	Y	W	CONS
PX	100		-100	
PY		100	-100	
PW			200	-200
PL	-25	-75		100
PK	-75	-25		100

Designate part of the capital in each sector as fixed in that sector

Markets	Production Sectors			Consumers
	X	Y	W	CONS
PX	100		-100	
PY		100	-100	
PW			200	-200
PL	-25	-75		100
PK	-50	-15		65
PKX	-25			25
PKY		-10		10

\$OFFTEXT

PARAMETERS

TX;

TX = 0;

\$ONTEXT

\$MODEL:M24

\$SECTORS:

X ! Activity level for sector X
Y ! Activity level for sector Y
W ! Activity level for sector W (Hicksian welfare index)

\$COMMODITIES:

PW ! Price index for welfare (expenditure function)
PX ! Price index for commodity X
PY ! Price index for commodity Y
PL ! Price index for primary factor L
PK ! Price index for (mobile) capital
PKX ! Price index for sector-specific input for sector X
PKY ! Price index for sector-specific input for sector Y

```

$CONSUMERS:
    CONS      ! Income level for consumer CONS

$PROD:X s:1
    O:PX      Q:100
    I:PL      Q: 25  A:CONS T:TX
    I:PK      Q: 50  A:CONS T:TX
    I:PKX     Q: 25  A:CONS T:TX

$PROD:Y s:1
    O:PY      Q:100
    I:PL      Q: 75
    I:PK      Q: 15
    I:PKY     Q: 10

$PROD:W s:1
    O:PW      Q:200
    I:PX      Q:100
    I:PY      Q:100

$DEMAND:CONS
    D:PW      Q:200
    E:PL      Q:100
    E:PK      Q: 65
    E:PKX     Q: 25
    E:PKY     Q: 10

$OFFTEXT
$SYSINCLUDE mpsgeset M24

$INCLUDE M24.GEN
SOLVE M24 USING MCP;

*      Solve a counterfactual:

TX = 0.5;
$INCLUDE M24.GEN
SOLVE M24 USING MCP;

```

Exercise:

As an exercise after examining this model, try to guess how the introduction of the specific factors affects the responsiveness (elasticity) of X and Y outputs to the 100% tax. Run the model and compare it to the 50% X-sector tax results in model M21 (M1_MPS).

Model 25 use of an initially slack activity (e.g., modeling tax avoidance)

As noted in chapter one, an attractive and powerful feature of MPS/GE is that it solves complementarity problems in which some production activities can be slack for some values of parameters and active for others. This allows researchers to consider a much wider set of problems that is allowed using software which can only solve systems of equations.

Model 25 presents a simple example, motivated by tax evasion activities. There is a third sector, Z, which also produces good X but it is 10% less efficient (10% more costly) than the X activity itself. So initially, Z does not operate. But when a tax of 25% is imposed on X, this activity goes slack and Z begins to operate. We could think of Z as a tax evasion or “informal” activity that is less efficient but can successfully avoid the tax.

\$TITLE Model M25: Closed 2x2 Economy with an Unprofitable Activity

\$ONTEXT

Markets	Production Sectors			Consumers
	X	Y	W	CONS
PX	100		-100	
PY		100	-100	
PW			200	-200
PL	-40	-60		100
PK	-60	-40		100

Activity Z is unprofitable at initial equilibrium prices. It is therefore not operated, and we cannot infer its technical properties from the benchmark social accounting data. We assume that Z is 10% less efficient than X.

\$OFFTEXT

PARAMETERS

TX;

TX = 0;

\$ONTEXT

\$MODEL:M25

\$SECTORS:

X ! Activity level for sector X
 Y ! Activity level for sector Y
 W ! Activity level for sector W (Hicksian welfare index)
 Z ! Alternative activity for producing X.

\$COMMODITIES:

PX ! Price index for commodity X
 PY ! Price index for commodity Y
 PL ! Price index for primary factor L
 PK ! Price index for primary factor K
 PW ! Price index for welfare (expenditure function)

\$CONSUMERS:

CONS ! Income level for consumer CONS

\$PROD:X s:1

O:PX Q:100
 I:PL Q: 40 A:CONS T:TX
 I:PK Q: 60 A:CONS T:TX

\$PROD:Y s:1

O:PY Q:100
 I:PL Q: 60
 I:PK Q: 40

\$PROD:Z s:1

O:PX Q:100
 I:PL Q: 44
 I:PK Q: 66

\$PROD:W s:1

O:PW Q:200
 I:PX Q:100
 I:PY Q:100

\$DEMAND:CONS

D:PW Q:200
 E:PL Q:100
 E:PK Q:100

\$OFFTEXT

\$SYSINCLUDE mpsgeset M25

PW.FX = 1;

Z.L = 0;


```
$INCLUDE M25.GEN
SOLVE M25 USINCP MCP;
```

* Let's levy a high on sector X and see what happens:

```
TX = 0.25;
$INCLUDE M25.GEN
SOLVE M25 USING MCP;
```

* What is the effect of the tax if Z could not be used?

```
Z.FX = 0;
```

```
TX = 0.25;
$INCLUDE M25.GEN
SOLVE M25 USING MCP;
```

Exercise:

As a second counterfactual, we impose the tax but do not let the Z sector operate by imposing the restriction $Z.FX = 0$;

(1) Compare the results of this run to the first counterfactual in which Z is allowed to operate. Can you interpret the welfare results? Hint: while the tax is distortionary, the switch to the inefficient activity uses real resources to avoid the tax.

(2) Now raise the tax to 100%. Does this result that tax evasion (the switch to Z) is welfare worsening still hold?

Model 26 introduces a labor supply or labor/leisure choice activity

Often general-equilibrium models used in international trade assume that factors of production, especially labor, are in fixed and inelastic supply. But designing tax, welfare, and education systems, endogenizing labor supply is a crucial part of the story. Model M26 endogenizes labor supply, allowing labor to choose between leisure and labor supply with leisure entering into the workers utility function.

This requires the modeler to specify an endowment of labor or time, something which is not in itself directly observable. Only the portion actually supplied to the market is observable. The modeler also needs to specify an elasticity of substitution between leisure and consumption goods, which will in turn imply an elasticity of labor supply.

We cannot do much theoretical analysis here, but do suggest that students interested in these questions work with a simple CES function with one good and leisure in order to understand the basic microeconomics of labor supply. For a Cobb-Douglas function with an elasticity of substitution equal to one between consumption and leisure, labor supply is completely inelastic with respect to the wage rate. When the elasticity of substitution is greater than one, an increase in the wage rate will mean an increase in labor supply, and an elasticity of substitution less than one will mean that labor supply is “backward bending”, falling with an increase in the wage rate.

In our formulation, we introduce an additional activity T, which transforms leisure (price PL) into labor supplied (price PLS). Strictly speaking this is not necessary, the consumer’s endowment of labor could just be supplied to both production and the welfare generating activity W. But adding activities is often useful. The solution will report the activity level of T which allows us to directly check the change in labor supply, and any tax on labor can be specified here just once rather than in each sector that uses labor. Here is the production block for T.

```
$PROD:T
O:PLS Q:100
I:PL Q:100 A:CONS T:TL
```

The production block for W specifies a nesting structure in which goods have an elasticity of substitution of 1 between them in a lower nest, and goods and leisure have an elasticity of substitution between them of 0.5.

```
$PROD:W s:0.5 cons:1
O:PW Q:300
I:PX Q:100 cons:
I:PY Q:100 cons:
I:PL Q:100
```

The consumer is assumed to be endowed with 200 units of labor/leisure (found in the DEMAND block), of which 100 units are supplied to the labor market initially.

\$TITLE Model M26: 2x2 Economy with Labor-Leisure Choice

\$ONTEXT

Activity T transforms leisure into labor supply:

Markets	Production Sectors				T	Consumers
	A	B	W	CONS		
PX	80	20	-100			
PY	20	80	-100			
PW			300			-300
PLS	-40	-60		100		
PL			-100	-100		200
PK	-60	-40				100

\$OFFTEXT

PARAMETERS

TL

WELFARE

REALCONS;

TL = 0;

\$ONTEXT

\$MODEL:M26

\$SECTORS:

X ! Activity level for sector X
 Y ! Activity level for sector Y
 T ! Labor supply
 W ! Activity level for sector W (Hicksian welfare index)

\$COMMODITIES:

PX ! Price index for commodity X
 PY ! Price index for commodity Y
 PL ! Price index for leisure
 PLS ! Price index for labor supply (factor L input)
 PK ! Price index for primary factor K
 PW ! Price index for welfare (expenditure function)

\$CONSUMERS:

CONS ! Income level for consumer CONS

```

$PROD:X s:1
      O:PX   Q:100
      I:PLS  Q: 40
      I:PK   Q: 60

$PROD:Y s:1
      O:PY   Q:100
      I:PLS  Q: 60
      I:PK   Q: 40

$PROD:T
      O:PLS  Q:100
      I:PL   Q:100  A:CONS T:TL

$PROD:W s:0.5  cons:1
      O:PW   Q:300
      I:PX   Q:100  cons:
      I:PY   Q:100  cons:
      I:PL   Q:100

$DEMAND:CONS
      D:PW   Q:300
      E:PL   Q:200
      E:PK   Q:100

$OFFTEXT
$SYSINCLUDE mpsgeset M26

PW.FX = 1;

$INCLUDE M26.GEN
SOLVE M26 USING MCP;

WELFARE = W.L;
REALCONS = (PX.L*X.L*100 + PY.L*Y.L*100)/(PX.L**0.5*PY.L**0.5*200);
DISPLAY WELFARE, REALCONS;

*      Solve a counter-factual, tax labor supply at 25%

TL = 0.5;
$INCLUDE M26.GEN
SOLVE M26 USING MCP;

WELFARE = W.L;
REALCONS = (PX.L*X.L*100 + PY.L*Y.L*100)/(PX.L**0.5*PY.L**0.5*200);
DISPLAY WELFARE, REALCONS;

```

Several features of this program require some explanation. First, the use of the labor supply activity is going to imply two separate prices in the presence of the labor tax. There is PL, which is the price of leisure, or alternatively the consumer's price of a labor supplied to the market. PLS is going to be the producer's cost of labor. The two are related by

$$PLS = PL(1+TL)$$

We could also refer to these as the before (PL) and after tax wage (PLS). We will spend more time on this issue in the next chapter and are simply alerting the reader to this issue now: in the presence of taxes, generally not all prices can be one, and it is very important to keep track of what prices go where, who pays what price and who receives what price.

Second, note the equations and notation at the end of the program.

```
WELFARE = W.L;
REALCONS = (PX.L*X.L*100 + PY.L*Y.L*100)
            / (PX.L**0.5*PY.L**0.5*200);
DISPLAY WELFARE, REALCONS;
```

A “.L” after a variable asks for the current value of a variable. Thus “W.L” gives the value of the variable named “W”.

In the first two of these statements, parameters (declared earlier) are assigned values following the solution to the model. The first is just the value of welfare. The second, “REALCONS”, is short for the real value of goods consumption. This is specified as the value of X plus Y consumption divided by the price index for a unit of welfare (the exact consumer price index). We have specified this variable (declared as a “parameter” in GAMS) in order to make an economic point.

As you will see if you run this model, the labor tax leads to a reduction in labor supply. This of course leads to a fall in commodity consumption but also to a rise in leisure. Economic indices generally ignore leisure and report changes in nominal or real consumption. If you look at the solution to the model, you will see that indeed REALCONS falls much more than WELFARE. Thus in this case, the usual statistics overstate the burden of the tax and would overstate the benefit of removing the tax if labor supply increases.

Finally, GAMS does not automatically write out the values of parameters in the listing file (in this case M26.LST). You have to request that, which is done here with the DISPLAY statement as shown. Parameter values will be written out following the values of the variables of the model.

Exercise:

Change the elasticity of substitution between leisure and goods and see how this affects the results concerning the labor tax.

Model M27 two forms of labor supply, such as to formal/informal sectors

This is a model which may be of interest to development and public finance economists. It assumes that there are two labor markets, a “formal” and an “informal” market. Governments are able to collect taxes on the former but not on the latter. The representative household can choose how much labor to supply to each market. For simplicity, we assume that there is no labor-leisure decision, and that all labor is supplied to one of the two markets, but that can be very easily added and indeed we will suggest that as an exercise at the end.

There are many ways of doing this. We first of all use an activity denoted LS which takes household labor and produces two outputs, formal and informal labor (prices PLSF and PLSI) according to a CET transformation function with an elasticity 5.0. Think of this as a household technology embodying the fact that the two types of labor are not perfect substitutes in supply. For example, this might be a crude simplification of the fact that the representative household is actual many households (or household locations) some of whom are better at supplying formal labor and vice versa. Concentrating supply in either market leads to something like “diminishing returns”, a concave transformation frontier between the two types of labor.

In addition, the two types of labor can be imperfect substitutes on the production side. That is what we assume here. Only formal labor is used in the X sector, while both formal and (mostly) informal labor are used in the Y sector. The two types of labor are in a lower level nest with an elasticity of substitution of 3.

The formal and informal labor supplies could go directly into production, but the listing file will not directly tell us how much of each type is produced by activity LS. Thus we use two “dummy” activities LF and LI which take a unit of formal labor (LF) or informal labor (LI) and just turn each unit into a unit of the same thing with a different commodity name (PLF and PLI) which are the actual inputs into production. This is just a trick so that the listing file will tell us how much of each type of labor is supplied, the activity levels of LF and LI respectively. In addition, this trick is convenient in multi-sector models because the tax on formal labor need only be specified once, in the LF activity, and not in every production block using formal labor.

Here is the program. You will see from the listing file that the tax on formal labor supply leads to a large shift of household supply toward informal labor and that there is a large shift in output toward Y, the sector using informal labor.

\$TITLE Model M27: 2x2 Economy with Formal/Informal Labor Supply

\$ONTEXT

Activity LS transforms leisure into formal and informal labor supplies. LF and LI are "dummy" activities used to keep track of how much labor is supplied to each market.

Markets	Production Sectors			Consumers			
	X	Y	W	LF	LI	LS	CONS
PX	100		-100				
PY		100	-100				
PW			200				-200
PLF	-40	-10		50			
PLI		-50			50		
PLSF				-50		50	
PLSI					-50	50	
PL						-100	100
PK	-60	-40					100

\$OFFTEXT

PARAMETERS

TL;

TL = 0;

\$ONTEXT

\$MODEL:M27

\$SECTORS:

X ! Activity level for sector X
 Y ! Activity level for sector Y
 LS ! Activity level for household labor supply
 LF ! Activity for formal labor supply
 LI ! Activity for informal labor supply
 W ! Activity level for sector W (Hicksian welfare index)

\$COMMODITIES:

PX ! Price index for commodity X
 PY ! Price index for commodity Y
 PL ! Price index for labor
 PLSF ! Price index for formal labor supplied to market
 PLSI ! Price index for informal labor supply to market
 PLF ! Price index for formal labor supplied to firms

PLI ! Price index for informal labor supplied to firms
 PK ! Price index for primary factor K
 PW ! Price index for welfare (expenditure function)

\$CONSUMERS:

CONS ! Income level for consumer CONS

\$PROD:X s:1

O:PX Q:100
 I:PLF Q: 40
 I:PK Q: 60

\$PROD:Y s:1 a:3

O:PY Q:100
 I:PLF Q: 10 a:
 I:PLI Q: 50 a:
 I:PK Q: 40

\$PROD:LS t:5.0

O:PLSF Q: 50
 O:PLSI Q: 50
 I:PL Q:100

\$PROD:LF

O:PLF Q: 50
 I:PLSF Q: 50 A:CONS T:TL

\$PROD:LI

O:PLI Q: 50
 I:PLSI Q: 50

\$PROD:W s:1.0

O:PW Q:200
 I:PX Q:100
 I:PY Q:100

\$DEMAND:CONS

D:PW Q:200
 E:PL Q:100
 E:PK Q:100

\$OFFTEXT

\$SYSINCLUDE mpsgeset M27

PW.FX = 1;

\$INCLUDE M27.GEN

SOLVE M27 USING MCP;

* Solve a counter-factual, tax formal labor supply at 50%

```
TL = 0.5;  
$INCLUDE M27.GEN  
SOLVE M27 USING MCP;
```

Model 28 two households with different preferences and endowments

A great many questions of interest to trade and public finance economists involve issues of distribution rather than or in addition to issues of aggregate welfare. Households (or “consumers” in MPS/GE) may differ in their preferences and more importantly in their sources of income (or their factor endowments). For both reasons, different taxes and other government policies affect different households in different ways. In addition to creating deadweight losses or aggregate benefits, tax change can also significantly redistribute income among households.

Adding multiple household types is a straightforward extension of our earlier models. In model M28, we allow for two households. Household A is relatively well endowed with labor, and also has a preference for good Y, which is the labor-intensive good. Household B is relatively well endowed with capital and has a relative preference for the capital intensive good X.

Our counterfactual experiment is to place a tax on the factor inputs to X, assigning half the revenue to each consumer. As you will guess, this tax lowers the welfare of household B. However, the redistribution effect outweighs the overall deadweight loss of the tax for household A, which is actually better off. This welfare gain is a combination of a redistribution in favor of capital, and a lowering of the relative consumer price of Y, the good favored by household A. Note that we chose labor as numeraire in this program, and the consumer price index (PWA, PWB) will differ for the two households.

\$TITLE Model M28GMS: 2x2 Economy with Two Household Types

\$ONTEXT

Two households: differ in preferences and in endowments

Markets	Production Sectors				Consumers	
	X	Y	WA	WB	A	B
PX	100		-40	-60		
PY		100	-60	-40		
PWA			100		-100	
PWB				100		-100
PL	-25	-75			90	10
PK	-75	-25			10	90

\$OFFTEXT

PARAMETERS

TX;

TX = 0;

\$ONTEXT

\$MODEL:M28

\$SECTORS:

X ! Activity level for sector X
 Y ! Activity level for sector Y
 WA ! Welfare index for consumer A
 WB ! Welfare index for consumer B

\$COMMODITIES:

PX ! Price index for commodity X
 PY ! Price index for commodity Y
 PL ! Price index for primary factor L
 PK ! Price index for primary factor K
 PWA ! Price index for consumer A welfare
 PWB ! Price index for consumer B welfare

\$CONSUMERS:

CONSA ! Income level for consumer A
 CONSB ! Income level for consumer B

* More than one tax may be levied on a single transaction with
 * revenues accruing to different agents. As specified here, the

* ad-valorem tax rate on inputs to sector X equals $2 \cdot TX$. Half
 * of the tax revenue accrues to A and half to B.

```
$PROD:X s:1
  O:PX      Q:100
  I:PL      Q: 25  A:CONSA  T:TX  A:CONSB  T:TX
  I:PK      Q: 75  A:CONSA  T:TX  A:CONSB  T:TX
```

```
$PROD:Y s:1
  O:PY      Q:100
  I:PL      Q: 75
  I:PK      Q: 25
```

```
$PROD:WA s:1
  O:PWA     Q:100
  I:PX      Q: 40
  I:PY      Q: 60
```

```
$PROD:WB s:1
  O:PWB     Q:100
  I:PX      Q: 60
  I:PY      Q: 40
```

```
$DEMAND:CONSA
  D:PWA     Q:100
  E:PL      Q: 90
  E:PK      Q: 10
```

```
$DEMAND:CONSB
  D:PWB     Q:100
  E:PL      Q: 10
  E:PK      Q: 90
```

```
$OFFTEXT
$SYSINCLUDE mpsgeset M28
```

```
PL.FX = 1;
```

```
$INCLUDE M28.GEN
SOLVE M28 USING MCP;
```

* Solve a counterfactual:

```
TX = 0.25;
$INCLUDE M28.GEN
SOLVE M28 USING MCP;
```

Exercises:

- (1) You might (correctly) guess that there is no way to redistribute the tax unevenly and make both households worse off. That would violate the first theorem of welfare economics. Try some alternative distributions to check on this. You will need to specify two different tax parameters, but they should continue to sum to $0.5 = 2 \cdot 0.25$.
- (1) Recalibrate the data so that the households have the same preferences. Running the experiment gives then a welfare effect due only to the change in factor prices following the imposition of the tax.

Model 29 introduces Stone-Geary (LES) preferences to model non-homothetic demand

No budget study we are aware of has ever suggested that consumer preferences are homogeneous. Households tend to spend a much higher proportion of their income on food at low incomes, for example, than at high incomes. Part of the reason that trade economists and applied general-equilibrium modelers are so fond of homogenous functions is that they are technically much easier to handle than non-homogeneous functions. As we noted earlier, any constant-returns CES function can be completely specified by a single vector of values of inputs and output quantities, and the relative prices of inputs at that point.

One alternative formulation is known as the Stone-Geary utility function, which in turn gives rise to the linear expenditure system (LES) of demand equations, the latter being popular in budget studies. The Stone-Geary utility function is just a Cobb-Douglas function with the origin displaced from zero. These displacements, if positive, are typically called “minimum consumption requirements”, meaning that the consumer gets no positive utility until these needs are met. Consider a simple case in which there is a minimum consumption requirement in X but not in Y. The minimum X consumption is denoted X^* . The utility function is

$$U = (X - X^*)^\alpha Y^{1-\alpha}$$

If we maximized this subject to the usual budget constraint with income I, the demand functions for X and Y would be:

$$X = X^* + \frac{\alpha(I - p_x X^*)}{p_x} \quad Y = \frac{(1 - \alpha)(I - p_x X^*)}{p_y} \quad p_x X + p_y Y = I$$

The first equation is rather intuitive in words. It says that you first purchase the minimum consumption requirements, and then you spend a constant fraction (α) of remaining income net of the minimum requirements on X. Further algebra would give us the budget share spent on X

and the income elasticity of demand for X.

$$\frac{p_x X}{I} = \frac{\alpha I + (1 - \alpha)p_x X^*}{I} \quad \frac{I}{X} \frac{dX}{dI} = \frac{\alpha I}{\alpha I + (1 - \alpha)p_x X^*}$$

The budget share spent on X falls with increases in income, asymptotically approaching α as income rises. The income elasticity of demand rises with income, asymptotically approaching 1.

Suppose that we want to calibrate our initial data to the assumption that the income elasticity of demand for X is initially equal to 0.75. If we solve the share equation (equal to 0.5) in the data and the income-elasticity equation (equal to 0.75 by assumption), we get $\alpha = 3/8$. This will then allow us to solve for X^* , which is $X^* = 40$. The trick is then to revise the benchmark data matrix, giving the consumer a negative endowment of $X = 40$. The utility (welfare) function W then has an input of 60 units of X (100 minus the minimum consumption requirement) and 100 units of Y. At prices of 1 for each good, MPS/GE will then calibrate the Cobb-Douglas utility function with an $\alpha = 3/8$ (60/160).

The counterfactual experiment in this model is to double the consumers endowment. Note from the results that there is a shift in consumption toward Y, the high income-elasticity good. Of course, the change in X and Y consumption cannot be directly interpreted as income elasticities of demand, since prices will change in general equilibrium. The price of Y will rise relative to X, and the price of the factor used intensively in Y will rise relative to the price of the other factor.

One final word of caution about this model and the use of Stone-Geary. Welfare changes have to be interpreted carefully, because utility is not linear (homogenous of degree 1) in income at constant prices. As you will see from the results of this simulation, utility (W) more than doubles as we double the endowment. Note by way of intuition that if income is so low that the consumer could just barely buy the minimum consumption requirement, then utility would be zero. Also, we should note that if income is too small to even buy X^* (in general equilibrium, the endowment is insufficient to produce X^*), the solver will crash and not compute a solution. These difficulties are troubling, but we are sure that good policy must consider the fact that really poor people consume very different bundles of goods than rich people, and this has probably more to do with non-homogeneity than them having homogeneous but different preferences.

\$TITLE Model M29: Closed 2x2 Economy -- Stone Geary (LES) Preferences

\$ONTEXT

The observed data is:

Markets	Production Sectors			Consumers
	X	Y	W	CONS
PX	100		-100	
PY		100	-100	
PW			200	-200
PL	-40	-60		100
PK	-60	-40		100

But calibrated to the model as:

Markets	Production Sectors			Consumers
	X	Y	W	CONS
PX	100		-60	-40
PY		100	-100	
PW			160	-160
PL	-40	-60		100
PK	-60	-40		100

\$OFFTEXT

PARAMETERS

ENDOW;

ENDOW = 1;

\$ONTEXT

\$MODEL:M29

\$SECTORS:

X ! Activity level for sector X
 Y ! Activity level for sector Y
 W ! Activity level for sector W (Hicksian welfare index)

\$COMMODITIES:

PX ! Price index for commodity X
 PY ! Price index for commodity Y
 PL ! Price index for primary factor L
 PK ! Price index for primary factor K
 PW ! Price index for welfare (expenditure function)

\$CONSUMERS:

CONS ! Income level for consumer CONS

\$PROD:X s:1

O:PX Q:100
 I:PL Q: 40
 I:PK Q: 60

\$PROD:Y s:1

O:PY Q:100
 I:PL Q: 60
 I:PK Q: 40

\$PROD:W s:1

O:PW Q:160
 I:PX Q: 60
 I:PY Q:100

\$DEMAND:CONS

D:PW Q:160
 E:PL Q:(100*ENDOW)
 E:PK Q:(100*ENDOW)
 E:PX Q:-40

\$OFFTEXT

\$SYSINCLUDE mpsgeset M29

PW.FX = 1;

\$INCLUDE M29.GEN

SOLVE M29 USING MCP;

* Counterfactual: double the factor endowment.

ENDOW = 2;

\$INCLUDE M29.GEN

SOLVE M29 USING MCP;