Lecture 2: Review of Consumer Theory, General Equilibrium

1. Equilibrium for the single consumer
   Utility function
   Budget Constraint
   Equilibrium

2. Market Demand
   The aggregation problem
   Identical homothetic utility
   Quasi-homotheticity
   Positive, normative interpretations of aggregate indifference curves

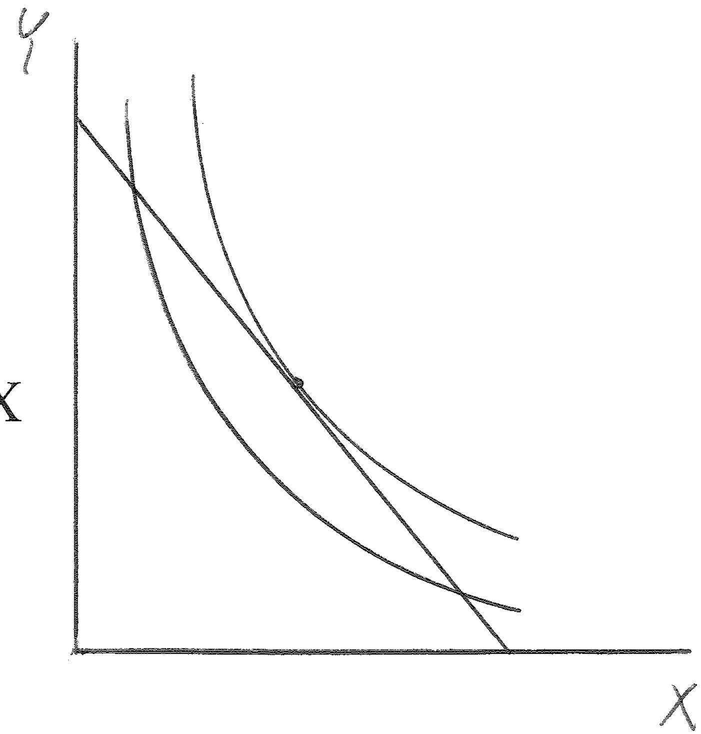
3. Equilibrium in the closed economy
   Equilibrium as the solution to an optimization problem
   Equilibrium as the solution to a system of inequalities

Consumer's Optimization Problem

We assume that the consumer maximizes his/her utility function subject to a budget constraint.

The budget constraint is given by:

$$I = p_x X + p_y Y$$



The slope of the budget constraint between Y and X

$$0 = p_x \Delta X + p_y \Delta Y \qquad \frac{\Delta X}{\Delta Y} = -\frac{p_x}{p_y}$$

If a consumer is optimizing, then the equilibrium will be characterized by a tangency between and indifference cure and the budget constraint.

$$MRS = \frac{p_x}{p_y}$$

Homogeneous Utility Functions

If U(X,Y) is homogeneous, then

the ratio of goods chosen in consumption, Y/X, depends only on the ratio of prices, p_x/p_y and not on income.

$$Y/X = F(p_x/p_y)$$

More formally, U is homogeneous of degree i if

$$U(\lambda X^0, \lambda Y^0) = \lambda^i U(X^0, Y^0)$$

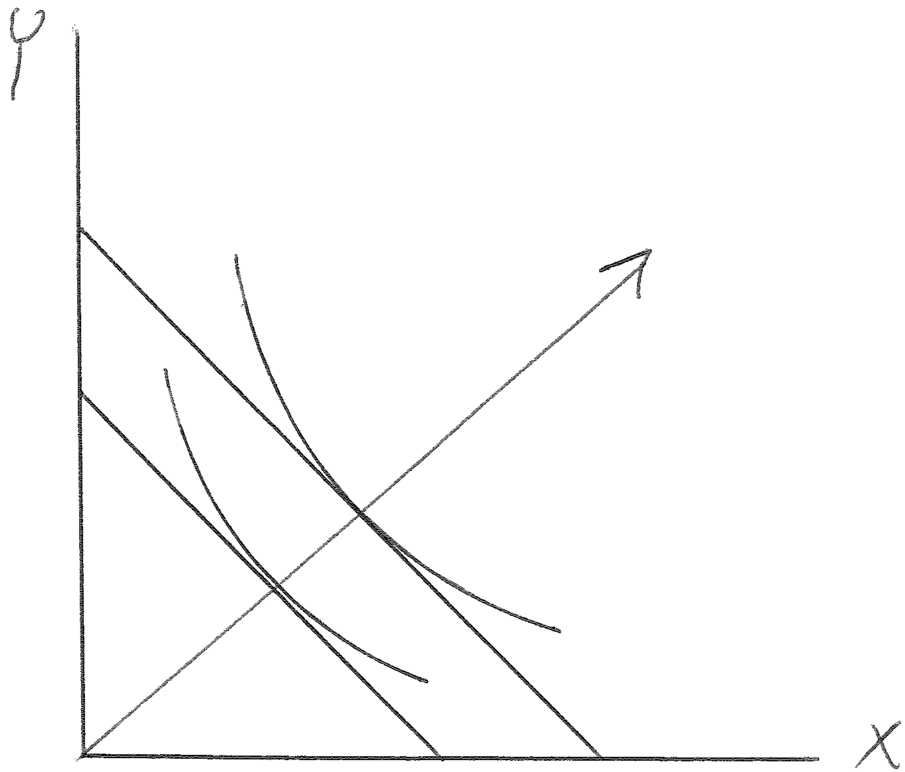
$$U_x(\lambda X^0, \lambda Y^0)\lambda dX = \lambda^i U_x(X^0, Y^0) dX$$

$$U_y(\lambda X^0, \lambda Y^0)\lambda dY = \lambda^i U_y(X^0, Y^0) dY$$

(1) The partial derivatives of a function which is homogeneous of degree i are homogeneous of degree i-1

$$\frac{U_x(\lambda X^0, \lambda Y^0)}{U_y(\lambda X^0, \lambda Y^0)} = \frac{U_x(X^0, Y^0)}{U_y(X^0, Y^0)}$$

(2) MRS depends only on X/Y ratio



$$U_x X^0 d\lambda + U_y Y^0 d\lambda = i\lambda^{i-1} U d\lambda \qquad \text{let } \lambda = 1, \text{ then}$$

$$U_x X^0 + U_y Y^0 = iU(X^0, Y^0)$$

(3) For a function HD1, the value of the function can be written as the sum of the partial derivatives each times the input level. I will use this result several times.

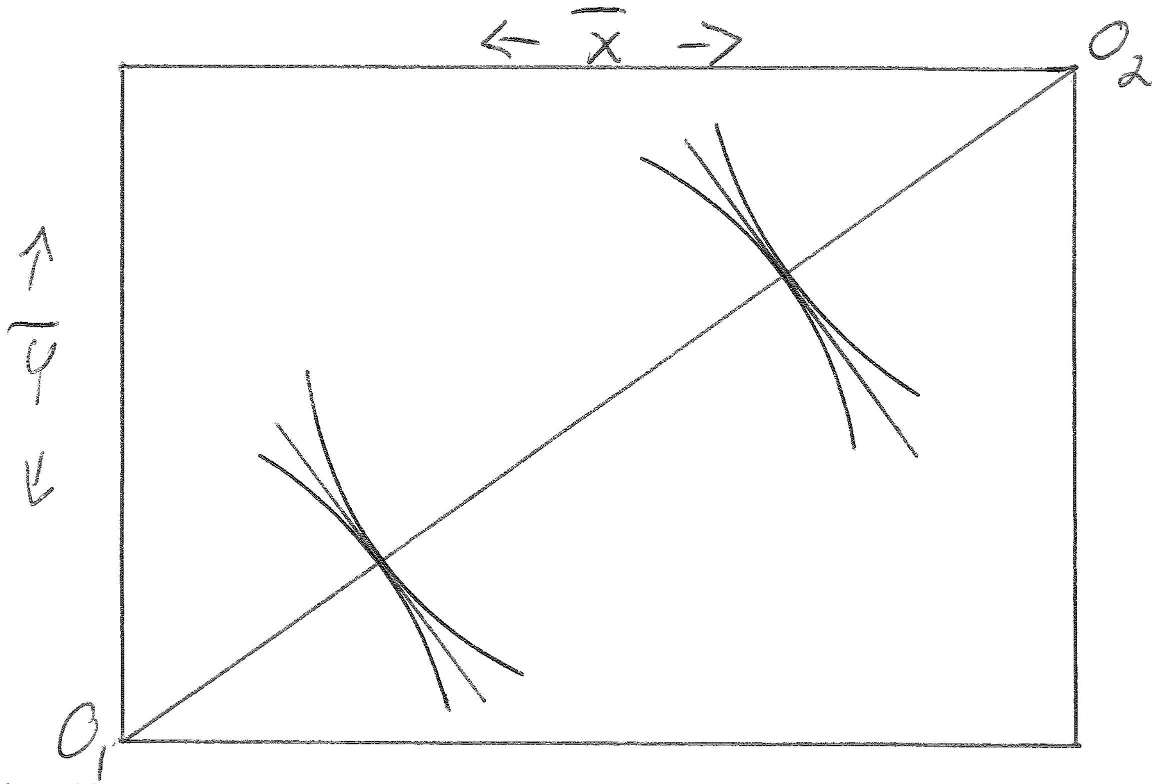
$$F(X_1, \ldots X_n) = \sum_i \frac{\partial F}{\partial X_i} X_i$$

Who cares? This assumption is used to "neutralize" differences in demand between countries as a cause of trade. Allows us to focus on differences in production as the cause of trade.

Pareto efficiency versus the distribution of welfare

An allocation of factors to production or an allocation of goods to consumers is said to be Pareto optimal, Pareto efficient, or just "efficient" if

There is no alternative allocation that makes on consumer better off without making another consumer worse off.



Both allocations A and B are Pareto efficient. However, they differ very much in the distribution of welfare between the two consumers.

In general, there are infinitely many Pareto efficient allocations, which differ in there distributional consequences.

Interpretation of "community" or national indifference curves.

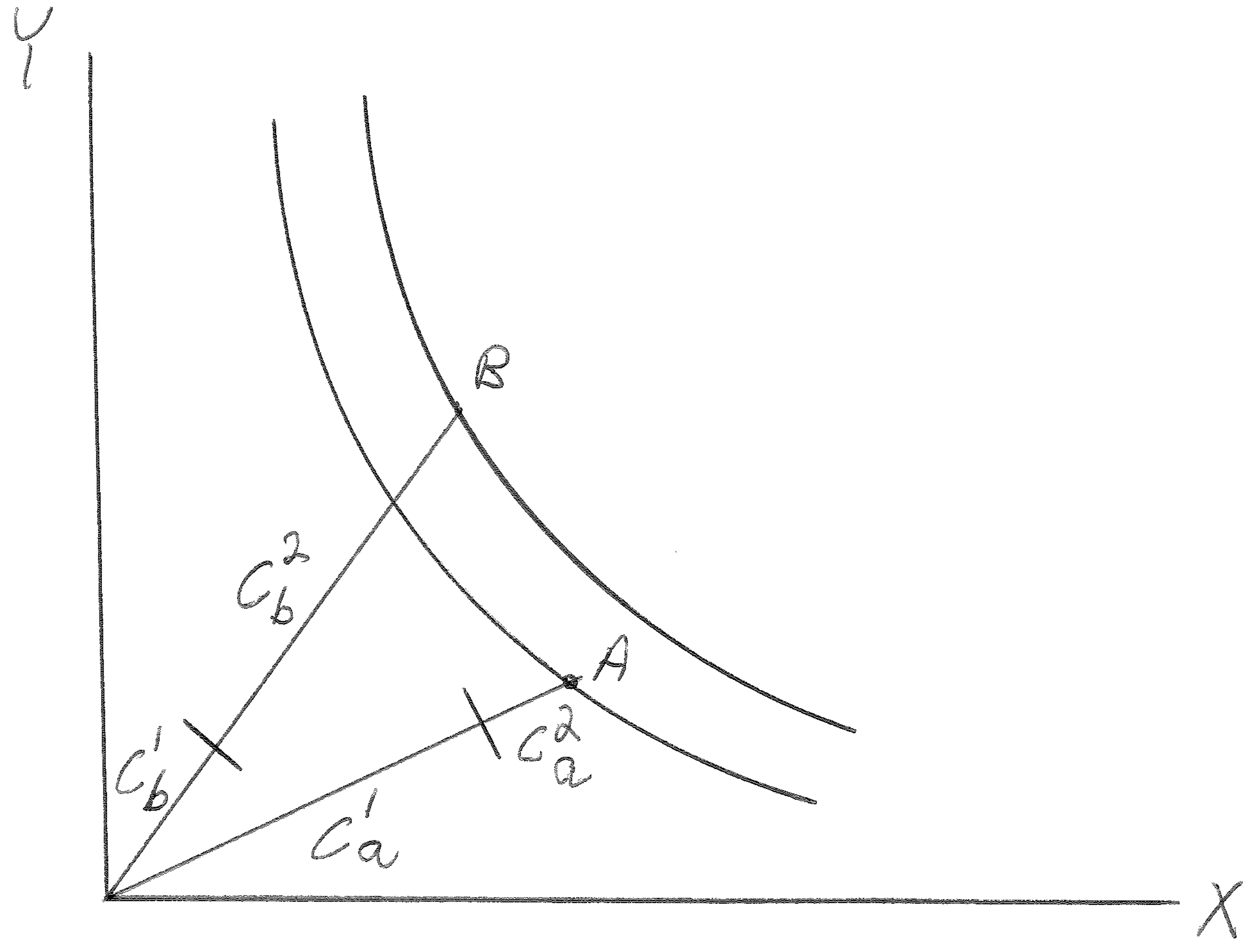
Positive statements: describe the facts: "what is"

Normative statements: describe the ideal or desirable: "what ought to be"

Community indifference curves.

Positive interpretation. CICs tell us what will be demanded at different price/national income combinations.

Normative interpretation. CICs are an index of national welfare: a higher curve is better.



So, we have to be careful about the normative interpretation of higher CICs: a higher curve does not mean that <u>everyone</u> is better off. The movement to a higher CIC is perfect consistent with someone being worse.

The Aggregation Problem

Can individuals with heterogeneous preferences and incomes be aggregated into a single national or "community" preference function?

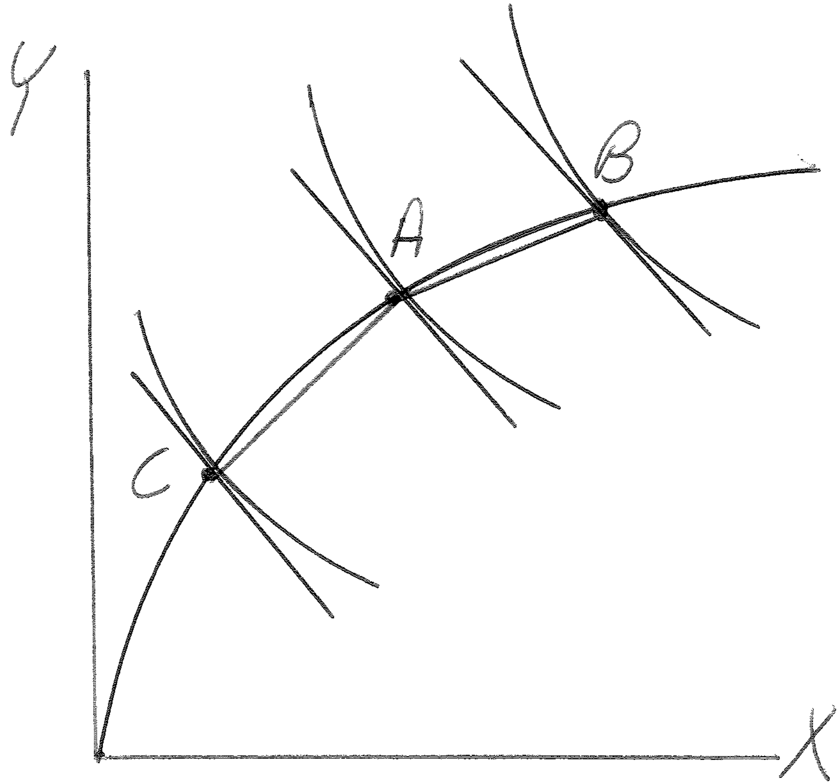
It turns out that this is issues is "almost" the same as the issue of whether or not demand for a good can be written as a function of price and aggregate income.

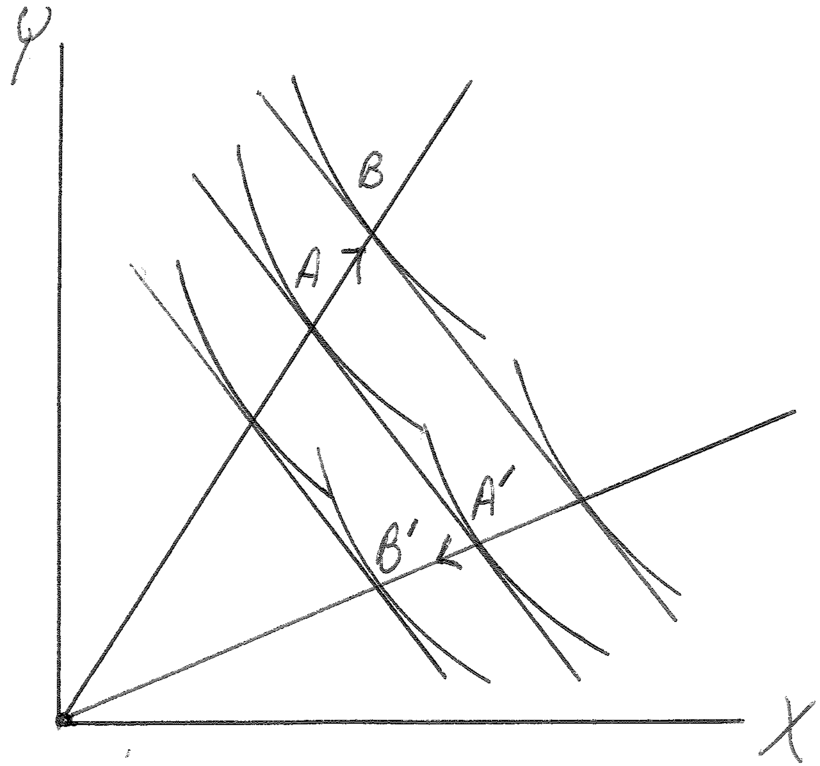
Let p be a vector of prices, and I1, ... , In be the incomes of n consumers. The question is whether or not we can write

$$D(p,\ I_1, \ldots, I_n) \ = \ D\left[p, \sum_{1}^{n} I_i\right]$$

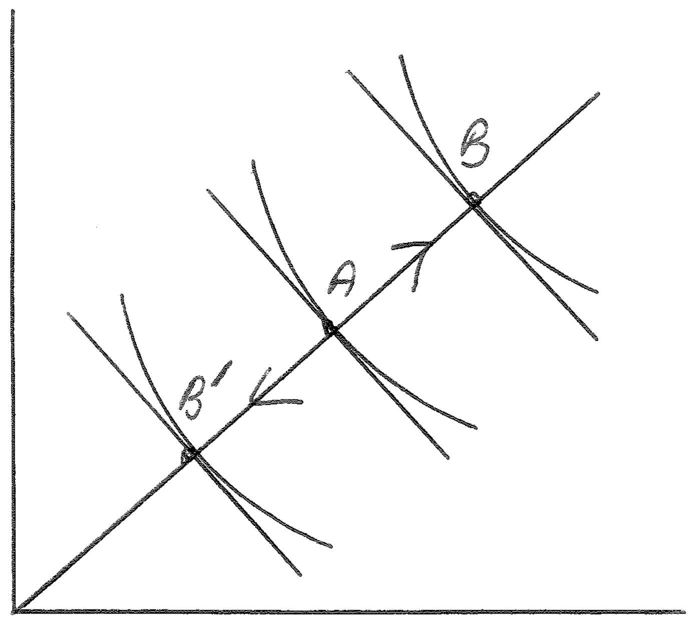
(1) Consumers have identical but non-homogeneous tastes

(2) Consumers have homogeneous, but non-identical tastes

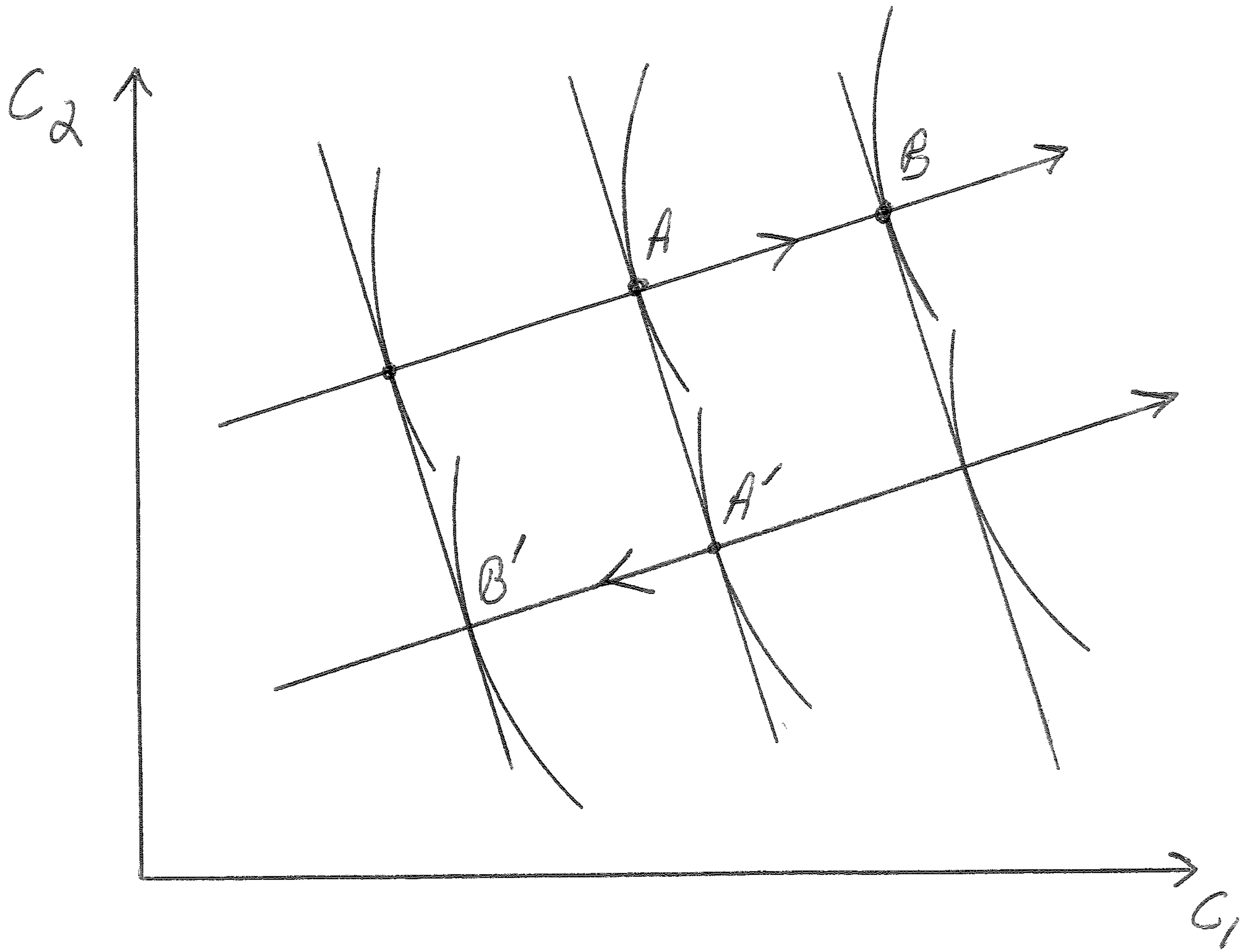




(3) Consumers have identical and homogeneous preferences



(1) Weakest possible assumption: Consumers have linear and parallel Engel's curves.



This in turn implies that consumer's have Stone-Geary utility functions: origin-shifted Cobb-Douglas, with identical share parameters for all consumers.

$$U_j = \prod_i^n \left[C_{ij} - C_{ij}^* \right]^{\alpha_i} \qquad C_{ij} = C_{ij}^* + \alpha_i \left[I - \sum_1^n p_i C_{ij}^* \right] / p_i$$

where the C^* are called "minimum consumption requirements

General Equilibrium in the Closed Economy

Three conditions characterize competitive equilibrium in a closed economy. Let X_c and X_p denote quantities of X consumed and produced respectively and similarly for Y.

(1) consumer optimization: $MRS = p_x/p_y$

(2) producer optimization: $MRT = p_x/p_y$

(3) market clearing: $X_c = X_p \quad Y_c = Y_p$

Note: market clearing implies that the budget constraint is satisfied

market clearing: value of prod = value of cons

zero profits: value of production = value of factor payments

consumer budget value of factor payments = value of consumption

How would we actually solve for general equilibrium? 2-good, 2-factor closed economy with fixed factor endowments, one representative consumer.

Take a very simply economy, two sectors (X and Y), two factors (L and K), and one representative consumer (utility function W). L and K are in inelastic supply. I is consumer income.

(1) $X = X(L_x, K_x)$ (2) $Y = Y(L_y, K_y)$

(3) $L^* = L_x + L_y$ (4) $K^* = K_x + K_y$

(5) $W = W(X, Y)$ (6) $I = wL^* + rK^* = p_x X + p_y Y$

This could be solved as a constrained optimization problem: Max (5) subject to (1), (2), (3), (4), and (6).

But this is awkward and messy if there are several consumer types and/or several countries. What do you maximize? The alternative approach is to convert the problem to a system of equations, and solve that system. First, solve the underlying cost minimization problems for producers and consumers, yielding

Unit cost functions for X and Y $\quad c_x = c_x(w,r), \quad c_y = c_y(w,r)$

Unit cost (expenditure) function for W $\quad e = e(p_x,p_y)$

Then specify the equilibrium as the solution to a system of 9 equations in 9 unknowns. The optimization problem is then converted to an economic equilibrium problem.

(1) Zero profit for X $\quad p_x = c_x(w,r)$

(2) Zero profit for Y $\quad p_y = c_y(w,r)$

(3) Zero "profit" for W $\quad p_w = e(p_x,p_y)$

(4) Supply = Demand for X	$X = e_{px}(p_x, p_y)W$
(5) Supply = Demand for Y	$Y = e_{py}(p_x, p_y)W$
(6) Supply = Demand for W	$I = e(p_x, p_y)W$
(7) Supply = Demand for L	$L^* = c_{xw}X + c_{yw}Y$
(8) Supply = Demand for K	$K^* = c_{xr}X + c_{yr}Y$
(9) Income Balance	$I = wL^* + rK^*$

These equations are of three types

- Product exhaustion
- Market clearing
- Income balance

Solve these nine equations for the unknowns

$X \;\; Y \;\; W \;\; p_x \; p_y \; p_w \;\; w \;\; r \; I$ (One equation redundant by Walras' Law)

Open Economy: suppose that we can trade X for Y at fixed world prices p*. Then the market-clearing conditions (c for consumption, p for production)

(4) Supply = Demand for X $\quad X_p = e_{px}(p_x, p_y)W = X_c$

(5) Supply = Demand for Y $\quad Y_p = e_{py}(p_x, p_y)W = Y_c$

are replaced by the much weaker restriction that:

(4') The value of the excess demand for X equals the value of the excess demand for Y (trade balance)

$$p_x^*(X_c - X_p) + p_y^*(Y_c - Y_p) = 0$$

or rearranged, the value of production must equal the value of consumption.

$$p_x^* X_p + p_y^* Y_p = p_x^* X_c + p_y^* Y_c$$

(4-5) automatically satisfy (4') but not vice versa.

General Equilibrium in the Open versus Closed (autarky) Economy