Review of Consumer Theory, Closed Economy and World Equilibrium: Chapter 3,4. (unotes2.pdf)

1. Maximizing utility subject to a budget constraint.
   p_x/p_y = MRS.

2. The assumption that the utility function is homogeneous.

3. Pareto optimality of an allocation versus the distribution of welfare.

4. Skip section 3.4 on aggregating preference.

5. Interpreting community indifference curves: positive description of demand versus normative description of welfare (section 3.5).

6. General Equilibrium in the closed economy. 2

Producer optimization
Consumer optimization
Market clearly
Pareto optimality of competitive equilibrium.

Chapter 4

We assume that the consumer maximizes his/her utility function subject to a budget constraint.

The budget constraint is given by:

$$I = p_1 X_1 + p_2 X_2$$

The slope of the budget constraint is the trade off between Y and X

$$0 = p_1 dX_1 + p_2 dX_2 \qquad \frac{dX_2}{dX_1} = -\frac{p_1}{p_2}$$

If a consumer is optimizing, then the equilibrium will be characterized by a tangency between and indifference cure and the budget constraint.

$$MRS = p_1 / p_2$$

Figure 3.1

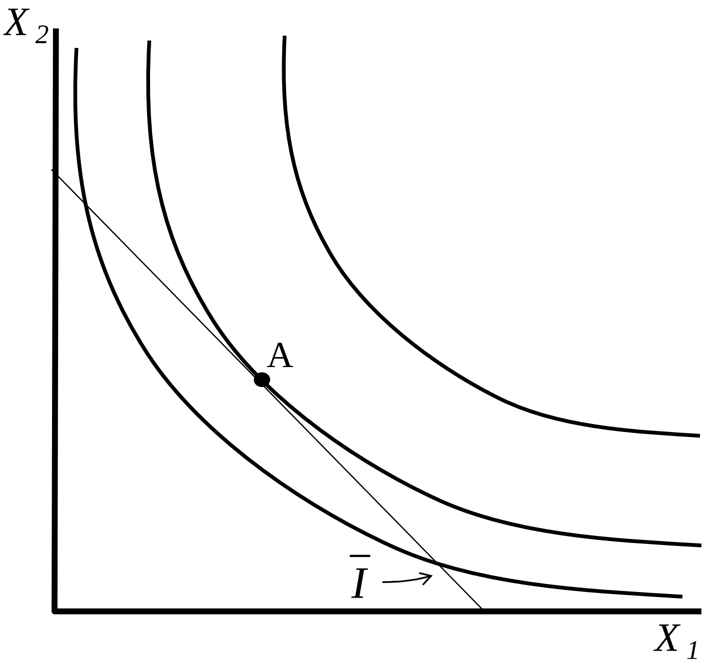
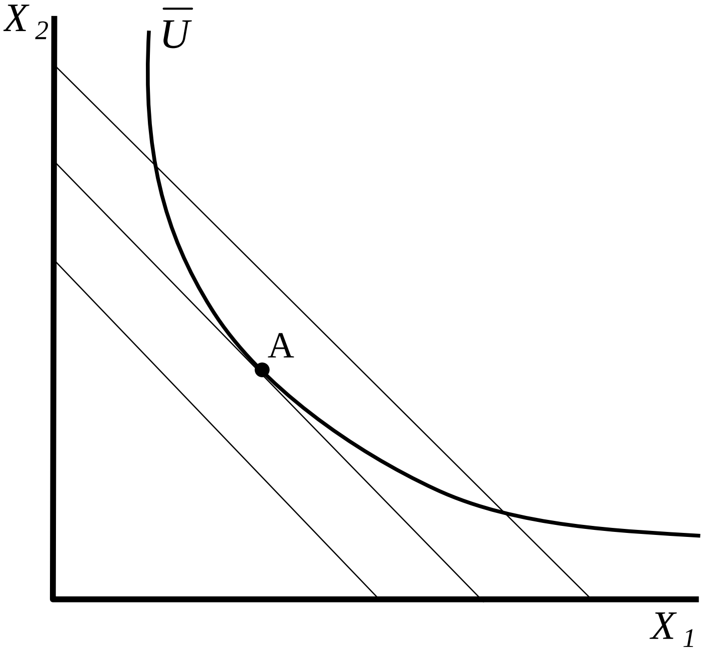


Figure 3.2



$$\max\ U(X_1, X_2) + \lambda(I - p_1X_1 - p_2X_2) \tag{3.1}$$

$$\frac{\partial U}{\partial X_1} - \lambda p_1 = 0 \qquad \frac{\partial U}{\partial X_2} - \lambda p_2 = 0 \tag{3.2}$$

$$I - p_1X_1 - p_2X_2 = 0 \tag{3.3}$$

Example: $U = X_1^{\beta} X_2^{1-\beta}$

Yields demand functions:

$$X_1 = \frac{\beta I}{p_1} \qquad X_2 = \frac{(1-\beta)I}{p_2} \tag{3.4}$$

Here is the definition of a homogeneous function:

If $U(\lambda X_1^0, \lambda X_2^0) = \lambda^n U(X_1^0, X_2^0)$

then U is homogeneous of degree n

Implication

If $U(X_1, X_2)$ is homogeneous, then

the ratio of goods chosen in consumption, X_2/X_1, depends only on the ratio of prices, p_1/p_2 and not on income.

$$X_2 /X_1 = F(p_1 /p_2)$$

Figure 3.3

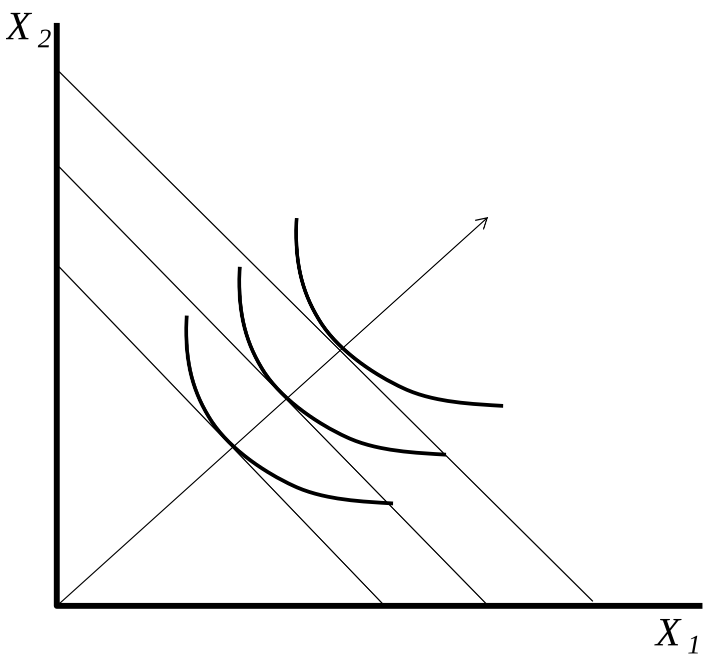


Figure 3.4

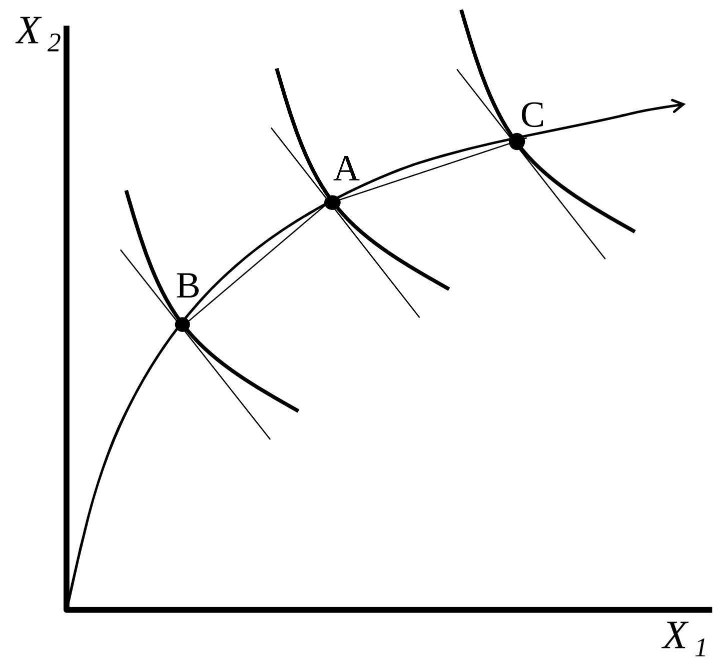


Figure 3.5

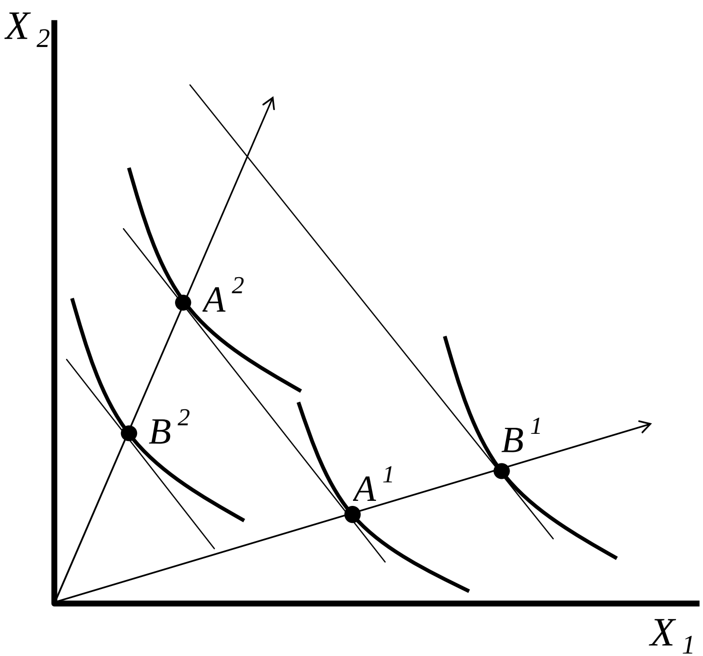
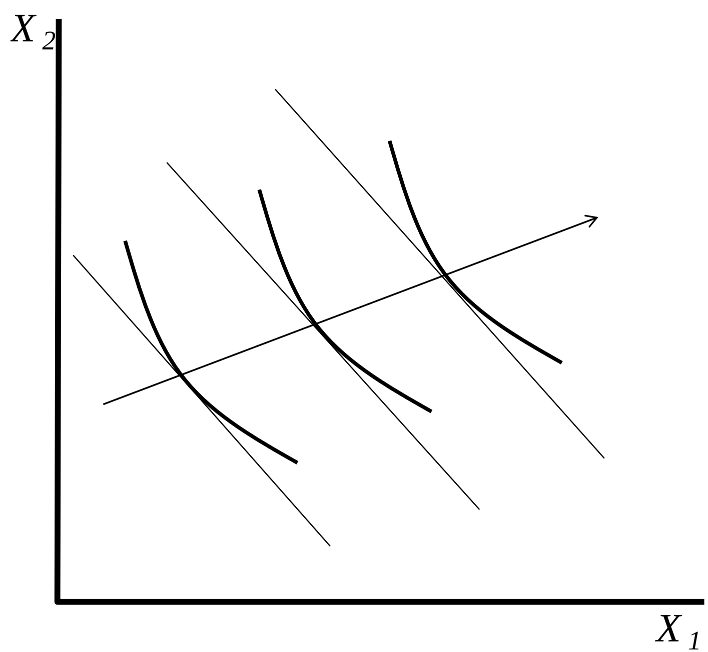


Figure 3.6



$$U(\lambda X^0, \lambda Y^0) = \lambda^i U(X^0, Y^0) \tag{3.10}$$

$$U_1(\lambda X_1^0, \lambda X_2^0)\lambda dX_1 = \lambda^i U_1(X_1^0, X_2^0) dX_1$$

$$U_2(\lambda X_1^0, \lambda X_2^0)\lambda dX_2 = \lambda^i U_2(X_1^0, X_2^0) dX_2 \tag{3.11}$$

$$\frac{U_2(\lambda X_1^0, \lambda X_1^0)}{U_2(\lambda X_1^0, \lambda X_2^0)} = \frac{U_1(X_1^0, X_2^0)}{U_2(X_1^0, X_2^0)} \tag{3.12}$$

Who cares? This assumption is used to "neutralize" differences in demand between countries as a cause of trade. Allows us to focus on differences in production as the cause of trade.

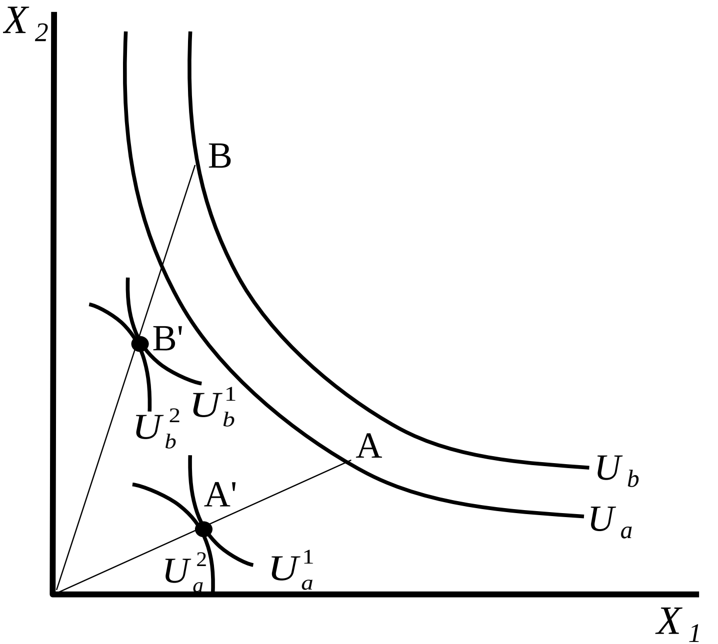
An allocation of factors to production or an allocation of goods to consumers is said to be Pareto optimal, Pareto efficient, or just "efficient" if

There is no alternative allocation that makes on consumer better off without making another consumer worse off.

Both allocations A and B are *Pareto efficient*. However, they differ very much in the *distribution* of welfare between the two consumers.

In general, there are infinitely many Pareto efficient allocations, which differ in their *distributional* consequences.

Figure 3.7



Interpretation of "community" or national indifference curves.

<u>Positive statements</u>: describe the facts: "what is"

<u>Normative statements</u>: describe the ideal or desirable: "what ought to be"

Community indifference curves.

<u>Positive interpretation</u>. CICs tell us what will be demanded at different price/national income combinations.

<u>Normative interpretation</u>. CICs are an index of national welfare: a higher curve is better.

=> ignores the fact that some households are generally worse off when a change leads to a higher index of national welfare.

Three conditions characterize competitive equilibrium in a closed economy.

Let D and X denote quantities of X consumed and produced respectively. Think of producers and consumers as facing exogenous prices.

(1) producer optimization: $MRT = p_1/p_2 = T(X_1, X_2)$

=> determines outputs, factor demands, factor prices => income

(2) consumer optimization subject to budget constraint (income from (1)):

$$MRS = p_1/p_2 = F(D_1, D_2) \text{ s.t. } I = p_1 D_1 + p_2 D_2$$

=> determines D_1, D_2

(3) market clearing: $D_1 = X_1 \quad D_2 = X_2$

=> determines p_1, p_2

market clearing: value of consumption = value of production

zero profits: value of production = value of factor payments

budget constraint: value of factor payments = value of consumption

Figure 4.1

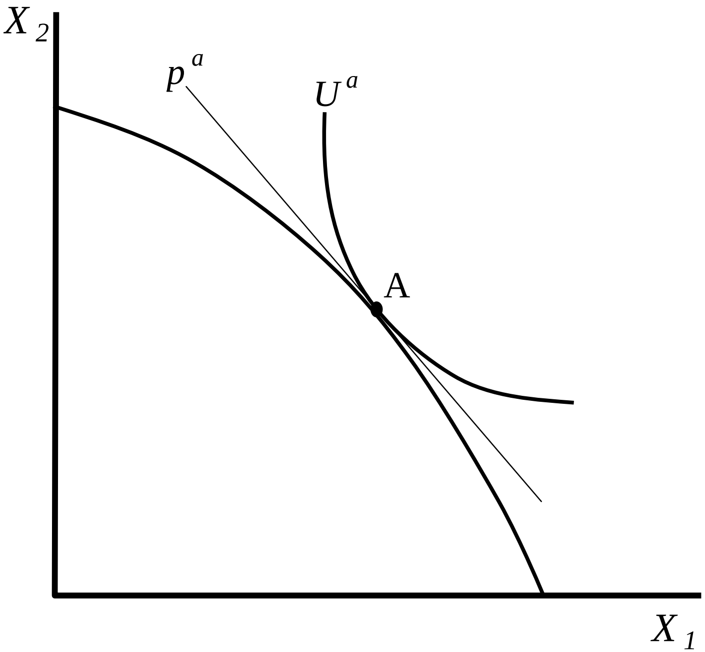
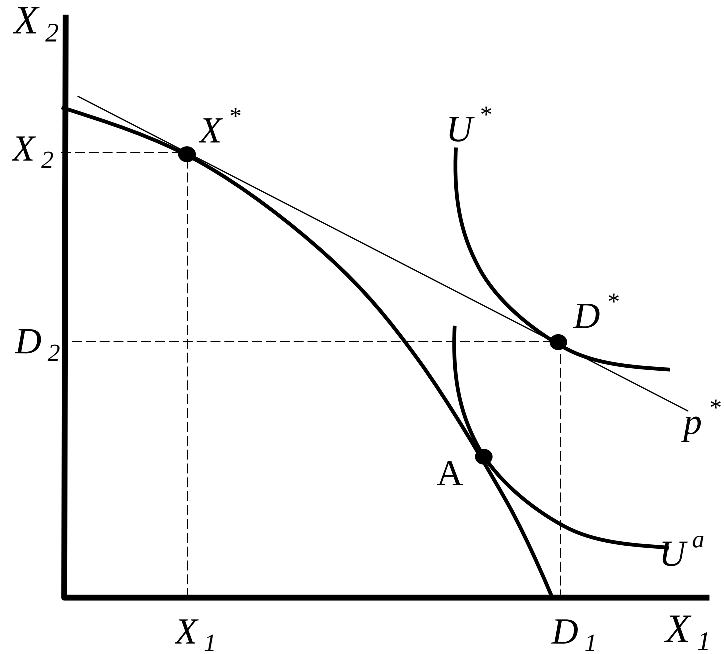


Figure 4.2



The benefit of trade is that the market-by-market clearing conditions $D = X \quad D_2 = X_2$ are replaced by something much less restrictive

Balance of trade constraint:

$$p_1^*(D_1 - X_1) + p_2^*(D_2 - X_2) = 0$$

This says the *value of imports of one good must be balanced by exports of the other good*.

This can also be written as:

$$p_1^* X_p + p_2^* Y_2 = p_x^* D + p_y^* D_2$$

This says that the *value of production must equal the value of consumption.*

The relationship between world and autarky prices determines the optimal *direction of trade*: which good is imported and which is exported.

The idea is simple: buy low, sell high. Buy from foreigners goods that are costly and difficult to produce at home and sell to foreigners goods that they value more highly than domestic consumers.

Figure 4.3

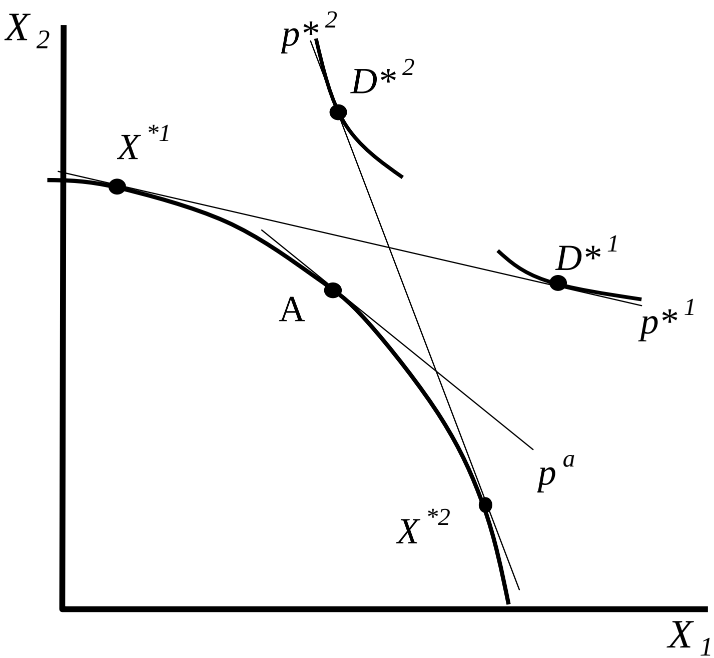
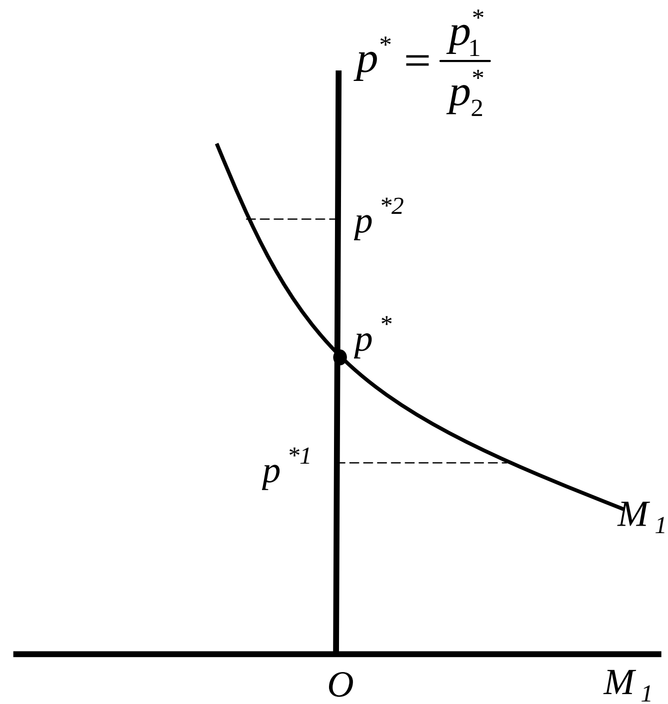


Figure 4.4



Final step: determining world equilibrium prices and world general equilibrium: in each market the desired exports of one country must equal the desired imports of the other country.

Construct an *excess demand curve* for each country: the consumption of a good minus production of that good at each world price.

Excess demand can be *negative*, meaning the good is exported or *positive*, meaning that it is imported.

In a two-good world, we only need to worry about one market by virtue of Walras' Law: if the world market for one good is in equilibrium and the each country is in trade balance (income/expenditure balance), then the other market must be in equilibrium.

If each country is in trade balance, then:

$$p_1^*(D_1^h - X_1^h) + p_2^*(D_2^h - X_2^h) = 0$$

$$p_1^*(D_1^f - X_1^f) + p_2^*(D_2^f - X_2^f) = 0$$

Add the equations together and rearrange:

$$p_1^*(D_1^h - X_1^h) + p_1^*(D_1^f - X_1^f) = -p_2^*(D_2^h - X_2^h) - p_2^*(D_2^f - X_2^f)$$

If the left-hand side is zero: there is world trade balance in the X market, then the right-hand side must also balance: there is trade balance in the world Y market.

So we can just consider the X market.

Figure 4.5

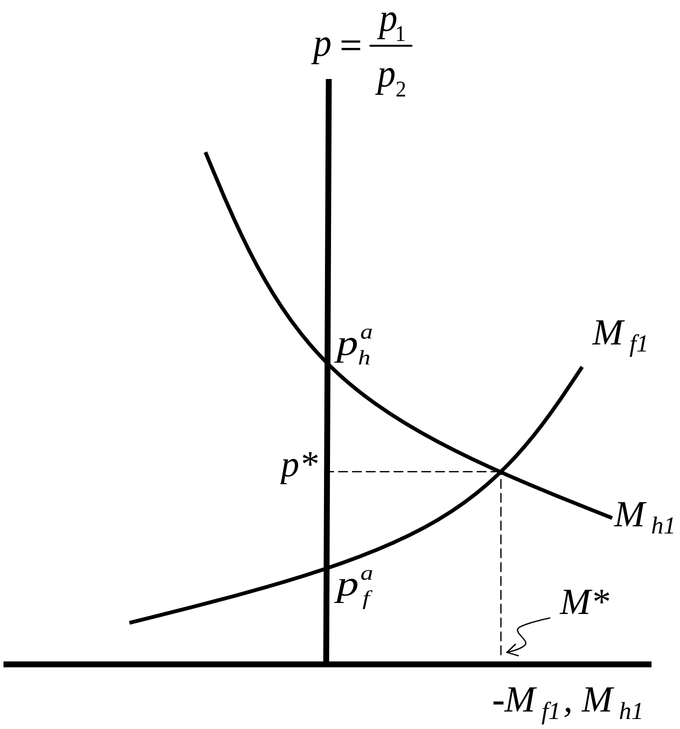


Figure 4.6

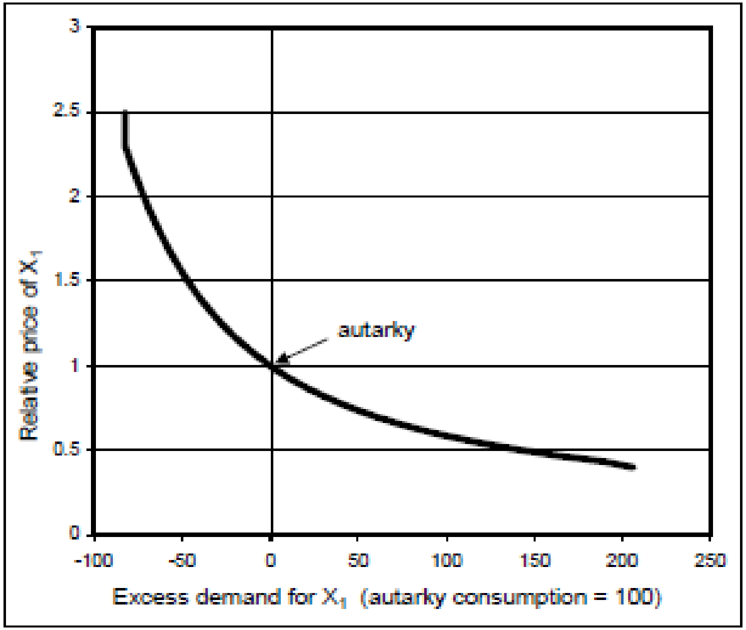


Figure 4.6: Simulated excess demand curve

Two-country (world) equilibrium is given at a price p* at which the desired exports of one country match the desired imports of the other country.

Note for future reference that the world price ratio must be between the autarky price ratios of the two countries.

If this was not true, then both countries would want to import the same good and export the same good and this cannot be an equilibrium.

Figure 4.7

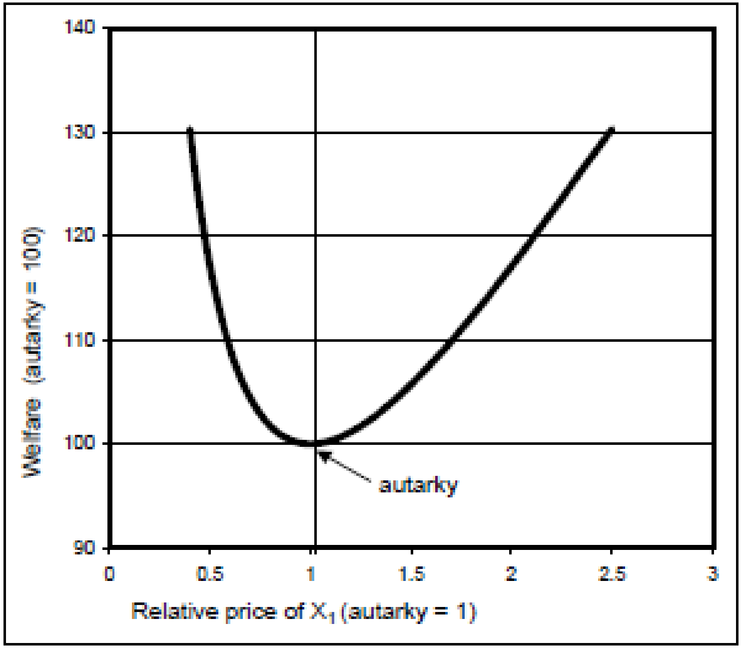


Figure 4.7: Welfare and the terms of trade

More advanced material: sophisticated version used in computer models:

First solve the individual producer and consumer optimization problems to get *cost functions* (usually call the *expenditure function* for consumers)

Unit Cost Functions: $c_x(w,r), \quad c_y(w,r), \quad e(p_x,p_y)$

Apply Shepard's Lemma: the derivative of a cost function with respect to an input price is the optimal quantity of that input used to produce one unit of output.

$$L_x = c_{xw}(w,r)X = \frac{\partial c_x(w,r)}{\partial w}X$$

Solving the general-equilibrium model: n equations in n unknowns

$c_x(w,r) = p_x$	X	zero-profits in X production
$c_y(w,r) = p_y$	Y	zero-profits in Y production
$e(p_x,p_y) = p_u$	U	zero-profits in U “production”
$X = e_{px}U$	p_x	supply = demand for X
$Y = e_{py}U$	p_y	supply = demand for Y
$L = c_{xw}X + c_{yw}Y$	w	supply = demand for labor
$K = c_{xr}X + c_{yr}Y$	r	supply = demand for capital
$e(p_x,p_y)U = (wL + rK)$	U	expenditure = Income