Putting Per-Capita Income back into Trade Theory and Policy

James R. Markusen
University of Colorado, Boulder

Abstract

A major role for per-capita income in international trade and trade policy, as opposed to simply country size, was persuasively advanced by many early economists including Linder (1961), Kuznets (1966), and Chenery and Syrquin (1975). Yet this crucial element of their story was abandon by most later trade economists in favor of the analytically-tractable but counter-empirical assumption that all countries share identical and homothetic preferences. This paper collects and unifies a number of disjoint points in the existing literature and adds several new insights using simple and tractable alternative preferences. Adding non-homothetic preferences to a traditional model helps explain such diverse phenomenon as a growing skill premium, the mystery of the missing trade, home bias in consumption, and the structural difference in growth due to rising per-capita income versus simply total income. With imperfect competition, we can explain higher markups and higher price levels in higher per-capita income countries, and the puzzle that gravity equations show a positive dependence of trade on per-capita-incomes, aggregate income held constant. A final section applies the model to policy by adding a global environmental good which has a high income-elasticity of demand. Results include an illustration that “issue linking” in trade negotiations, specifically linking trade and environment negotiations, is a productive way forward.

N.B., issues of product quality and the intra-country distribution of income, while of great importance and interest, have proved to be beyond the scope of one paper. A number of important papers are reference as an acknowledgment of valuable related work, but they are not analyzed or discussed.

This Draft, February July 20, 2012

Thanks to participants in many seminars, conferences and workshops, including the NBER trade group and the CEPR ERWIT conference, and seminars in Oxford, LSE, ETH Zurich, and Paris School of Economics. Special thanks to Adrian Wood, for gently reminding me that per-capita income was once an important part of international economics. Comments, suggestions and added citations welcome.

GAMS code for the simulations is available from the author on request.
1. Introduction

All international trade economists understand that many things can cause trade. However, our models and empirical analyses typically and appropriately tend to focus on one cause of trade at a time in order to understand how a particular basis for trade contributes to explaining trade patterns, determines gains from trade, and impacts on income distribution. That having been said, it seems that most trade theory focuses on production-side determinants of trade. It is typically assumed that consumers have identical and homothetic preferences within and across countries. Aggregate demand depends only on commodity prices and aggregate income, and it is independent of per-capita income. I also believe that it is appropriate to suggest that no one thinks that this is a good empirical assumption and that it is made for analytical convenience and tractability.¹

If we control for differences in prices across countries, the observation of different budget shares can indicate either that preferences differ and/or that they are non-homothetic. Two pure cases can be distinguished: one in which countries have homothetic but non-identical preferences and one in which countries have identical but non-homothetic preferences. I feel much more comfortable with the second alternative. Then demand differences are not only systematic but the hypothesis is testable and falsifiable.

The purpose of this paper is collect, synthesize, and new considerations to fragmented results from existing research in order to offer a generic model of identical but non-homothetic preferences and present a unified and testable set of results. In section two, the preferences are presented and analyzed and then placed on top of a standard two-good, two-factor, two-country Heckscher-Ohlin model. The resulting model offers alternative explanations for such diverse phenomenon as growing skill premiums, the mystery of the missing trade, home bias in consumption, and the important difference between growth in per-capita income and simply total income with per-capita income held constant. In section 3, I add scale economies and imperfect competition and show that the model can offer alternative explanations for higher price levels and higher markups in high-productivity economies, and a higher trade volume between identical high per-capita income countries, aggregate income held constant. Using the same model, it is noted that non-homothetic demand will imply that horizontal multinational firms are more concentrated among countries with similar per-capita income, not just similar total income.

Adding an environmental good with a high income-elasticity of demand provided additional results. This section illustrates how per-capita income influences a policy bargaining outcome, shows why an “equal sharing rule” (all countries agree to cut emissions by an equal proportion) may fail, and illustrates why linking trade and environmental issues is a good way forward.

¹A more egregious assumption is made in much of the strategic trade-policy literature: quasi-linear preferences in which the income elasticity of demand for increasing-returns good(s) is zero. Yet the industries offered as examples, such as aircraft and electronics are surely goods with income elasticities greater than one! The present author has of course been guilty himself of this atrocity.
A number of the ideas here are found in earlier papers focusing on specific issues. Several papers, obviously beginning with Linder, focused on monopolistic competition and the impact of non-homothetic preferences on intra versus inter-industry trade. Papers by Markusen (1986), Bergstrand (1990), and Francois and Kaplan (1996) draw out implications for intra-industry and total trade volumes. Matsuyama (2000) uses a competitive Ricardian model in which the South’s comparative-advantage goods are low-income-elasticity-of-demand goods to derive results similar to some here. Fieler (2011) uses a Ricardian approach ala Eaton and Kortum (2002), which yields outcomes related to those from monopolistic competition. There is a dispersion of technologies across goods and a variability of labor efficiency across countries. High income elasticity goods have a higher dispersion and are produced in high-income countries. This higher dispersion leads to more trade among the high-income countries relative to low-income countries. Recent working papers by Bernasconi (2009) and Martinez-Larzoso (2010) give strong empirical support to the positive relationship of bilateral trade to per-capita income, direct support for Linder.

Markusen (1986) is a three-region model with “east-west” trade among identical high-income (north) countries and “north-south” trade between the high and low per-capita-income countries. High income-elasticity goods are both capital-intensive and differentiated, while the homogeneous low-income elasticity good is labor intensive. The volume of east-west (north-north) trade is then higher than north-south trade relative to identical and homothetic preferences. Fieler (2011) makes substantial theoretical progress on this sort of multi-country prediction in her Ricardian model and deals with south-south trade as well.

Income-elasticity estimates for broad categories of consumption goods from Markusen and Hunter (1989) suggest income elasticities from about 0.45 (food) to 1.90 (medical care). Alternatively, non-homotheticity means that the shares of a country’s national expenditure for goods vary systematically with levels of per-capita income. Shares from Hunter (1991) and Cassing and Nishioka (2012) suggest that preferences are not just differing randomly across countries but observed shares are related to per-capita income.

Hunter (1991) shows that the influence of non-homotheticity is in the direction of reducing the volume of trade. A counter-factual analysis neutralizes the estimated non-homothetic and finds that the effect of imposing homotheticity is to raise trade flows by 29 percent. Cassing and Nishioka (2012) use a neutralization exercise similar to Hunter’s and find that developing countries consume relatively more labor-intensive goods than under preference homogeneity. Second, they find that preference biases between rich and poor countries explain a larger proportion of missing factor trade than do differences in technology, though preference differences are not distinguished from non-homotheticity. Fieler’s (2011) empirical results are strong and convincing. Her model correctly predicts both the large volume of trade among high-income countries and the low volume of trade among poor countries, whereas previous gravity-type work has done a poor job on the latter.

Bernasconi (2010) has an interesting alternative story, which is that high-per-capita-income consumers/countries consume a broader range of goods, and this in turn increases the
volume of north-north trade. Martinez-Zarzoso and Vollmer (2010) show a strong and positive relationship between trade volumes and per-capita income, and trade volumes and income inequality, with the latter relationship stronger for more sophisticated goods, both consistent with the model adopted below.

Some of these results contrast with Bowen, Leamer and Sveikauskas (1987) and Trefler (1995) who do introduce non-homothetic preferences into their analyses and get weak value added from doing so. Neither paper addresses non-homotheticity as a cause of missing trade or home bias (Trefler does find it helps solve the “endowment paradox”). Reimer and Hertel (2010) find that non-homothetic preferences play only a small role in missing trade over broad categories of goods. Rather, they find that, as income grows, it is directed toward the relatively capital-intensive version of a given good. But as will be noted, Caron, Fally and Markusen (2012) show that it is the skilled-labor intensity of a good, not it’s physical capital intensity, that is strongly correlated with the income elasticity of demand for that good.

Results in the section introducing imperfect competition are similar to those found in Wong (2003), Coibion, Einav and Hallak (2007), Hummels and Lugovskyy (2009) and Simonovska,(2009) which is that markups and hence the price level will be higher in the high per-capita-income country. Simonovska gets strong empirical support for this relationship. I should also note that Simonovska carefully considers identical products, which eliminates quality issues which could be an alternative explanation for systematic price differences by per-capita income. Essentially the same result was found by Wong for pricing of identical pharmaceutical products.

An area where per-capita income does play an important role is in the analysis of product quality. If a consumer is going to buy only one unit of a good or zero, then the quality demanded is likely to depend on per-capita income. This makes the average level of per-capita income important for trade but also the intra-country distribution of income matters for inter-country trade. I am the first to acknowledge that these issues are of great interest and importance. But it became clear to me that dealing with them is far beyond the scope of one paper for a number of reasons, including that they require a quite different analytical and empirical approach. Thus I will not deal with issues of product quality and/or the intra-country distribution of income, but include many references in a separate section at the end of the paper. A good place to start on this literature is a recent paper by Hallak (2010) which also notes the contributions of many earlier papers.

Many of the results (the policy section excepted) depend on a crucial assumption linking high ratios of skilled labor or capital intensity in production to high income elasticities of demand. In a recent paper (Caron, Fally and Markusen 2012), the authors find strong economic and statistical significance to this link; specifically, the correlation between goods’ skilled-labor intensity in production and their income elasticity of demand in consumption exceeds 50 percent.
2. A Generic Model

The preferences we will use are variation on a standard Stone-Geary utility function, to be introduced shortly. The production side of the model is deliberately Heckscher-Ohlin to permit an easy comparison with traditional results. There are two good (X and Y), two factors of production (K and L) and two countries home and foreign (h and f).

Throughout the paper, the following assumptions are made.

(1) good X is relatively capital (human or physical) intensive, and Y is relatively labor intensive
(2) good X has an income elasticity of demand greater than one
(3) good X is the increasing-returns good if there is one (section 3)
(4) the labor supply is identical to the number of households, implying that the capital-abundant country must be the high-per-capita-income country
(5) country h is relatively capital abundant when relative endowments differ
(6) country h has higher productivity when productivities differ across countries

Most of these assumptions are without loss of generality, but the intersection of (1) - (4) matters; in particular, that the capital-intensive good has the high income elasticity of demand. Empirical support for this assumption is found in Bergstrand (1990) and indirectly in Hunter (1991) Cassing and Nishioka (2012) and Caron, Fally and Markusen (2012). The latter find that the correlation between skilled-labor intensities of goods and their (estimated) income-elasticities in demand exceeds 50 percent, and find that this in turn implies that non-homothetic demand can explain about 1/3 of missing trade, explain trade volumes and trading partners, and by counterfactual-simulation explain a rising skill premium with neutral, world-wide productivity growth.

Matsuyama (2000) uses an equivalent assumption in his Ricardian model: the South’s comparative advantage goods are low-income-elasticity goods. Fieler’s (2011) theory is also Ricardian, and the theory and empirical evidence deliver a related result: the south has a comparative advantage in low-income elasticity standardized goods while the north has an advantage in high-income-elasticity goods whose production technologies are more variable across countries.

Since we will focus on a limited number of experiments, some short-hand terminology is used throughout. Productivity advantage or growth, or higher productivity refers to an equal proportional Hicks-neutral productivity advantage or growth in both sectors in one country (always country h). Factor accumulation refers to a equal proportional (‘neutral’) growth in the endowments of both factors of one or both countries: factor accumulation increases the number of households in the same proportion to total income.

Lower-case letters denote per-household quantities. In addition to x and y, there is a parameter z > 0 at the household level. Preferences or utility (u) are given as follows.
Virtually all the results go through with the CES-Cobb-Douglas version except perfect aggregation: aggregate demand always depends on the distribution of income. This function is hard to work with analytically (though easy for the computer).

We could interpret $z$ as an endowment good and assume that households cannot buy or sell $z$. $x$ could be televisions and $z$ could be watching a sunset (non-rivaled and non-excludable: sitting on a dock on the bay as Otis Redding might say). The assumption that $x$ and $z$ are additive has little to with the results of this paper, but has the advantages that (a) there is a simple analytical solution for demand and (b) aggregate demand does not depend on the distribution of income (with a qualification noted below).

It is more common to see Stone-Geary written with $(y - z)$ instead of $(x + z)$, with $z > 0$ then referred to as a “minimum consumption requirement”. But this leads to a problem if income is insufficient to purchase the minimum consumption requirement and no household will ever be observed to purchase only good $Y$. In addition, our formulation in (1) will mean that the price elasticity of demand for $X$ will be falling in per-capita income, a property exploited in Wong and in Simonovska.

Let $m^i$ denote the income of household $i$ and let $p_x$ and $p_y$ denote the prices of $X$ and $Y$. The households budget constraint is given by:

$$m^i = p_x x^i + p_y y^i$$  \hspace{1cm} (2)

Maximization of (1) subject to (2) gives the following Marshallian demand functions.

$$x^i = \max \left[ 0, (\beta - 1) z + \frac{\beta m^i}{p_x} \right] \quad y^i = \min \left[ \frac{m^i}{p_y}, \frac{(1 - \beta)(m^i + p_x z)}{p_y} \right]$$  \hspace{1cm} (3)

$$x^i > 0 \quad \text{iff} \quad m^i > \frac{(1 - \beta)p_x z}{\beta} = m^0$$  \hspace{1cm} (4)

At low levels of income, the household buys only good $Y$, and above the threshold income indicated in (4) by $m^0$, begins to buy $X$. This is an interesting and surely realistic point, and it makes aggregate demand depend on the distribution of income. I will assume for much of the paper that (4) holds with strict inequality for all households.

Let $L$ denote both the country’s labor supply and household measure so that $Z = zL$ denotes the economy-wide “endowment” of $Z$: $z$ is a parameter, while $Z$ is strictly proportional to the number of households. If (4) holds for all households, aggregate demand for $X$ is

2 Virtually all the results go through with the CES-Cobb-Douglas version

$$u = (x^{\alpha} + z^a)^{\beta/\alpha} y^{1 - \beta}$$

except perfect aggregation: aggregate demand always depends on the distribution of income. This function is hard to work with analytically (though easy for the computer).
The general n-good name for this type of demand function is “linear expenditure system” and is also used in Bowen, Leamer, and Sveikauskas, (1987) who refer to the $Z_s$ as “autonomous” expenditure. See Deaton and Muellbauer (1980) for the classic general analysis.

\[
X = \sum_{i=1}^{L} x^i = (\beta - 1)Z + \frac{\beta M}{p_x} \quad Z = zL \quad M = \sum_{i=1}^{L} m^i
\]  

Now consider the income elasticity of demand for $X$ and assume that income grows through a productivity increase, holding the number of households $L$ and therefore $Z$ constant.

\[
\left[ \frac{M}{X} \frac{dX}{dM} \right]_{Z=0} = \frac{\beta M}{\beta M + (\beta - 1)p_xZ} = \frac{m}{m - m^0} > 1
\]

(growth through productivity improvement)

If growth instead occurs through neutral factor accumulation, $Z$ is strictly proportional to $M$, then

\[
\left[ \frac{M}{X} \frac{dX}{dM} \right]_{Z=M} = 1 \quad \text{(growth through neutral factor accumulation)}
\]

Using \(dX/dp_x = -\beta M/p_x^2\), the Marshallian price elasticity, defined as positive, is

\[
\epsilon = -\left[ \frac{p_x}{X} \frac{dX}{dp} \right] = \frac{\beta M}{\beta M + (\beta - 1)p_xZ} = \frac{m}{m - m^0} > 1
\]

Thus the per-capita income and the price elasticities of demand for $X$ are (locally) the same.

Properties of the preferences are illustrated in Figure 1, using aggregate demands. The Engels curve (prices constant, number of households constant) is given by $0Y_0A$. Up to income $M_0$, given by (4) with equality, the household consumes only good $Y$ and then has a constant marginal propensity to consumer $X$ and $Y$ as income rises. At incomes above that which allows point $Y_0$ to be reached, the Engels curve is linear through $A$ at income level $M_0$ and reaching $B$ at income level $M_1$.

Consider point $A$ and income level $M_0$ in Figure 1. Now suppose instead we let the economy grow through proportional factor accumulation, adding households in strict proportion

---

\(^3\)The general n-good name for this type of demand function is “linear expenditure system” and is also used in Bowen, Leamer, and Sveikauskas, (1987) who refer to the $Z_s$ as “autonomous” expenditure. See Deaton and Muellbauer (1980) for the classic general analysis.
to the increase in income, so $Z$ and $M$ grow to $Z_f$ and $M_f$ respectively. Now the Engels curve beyond A will be given by a ray through the origin and points A and C and aggregate demand is homothetic with respect to aggregate income. Figure 1 gives the first important result of the paper: a growing economy will look very different depending on whether growth is through productivity or capital accumulation on the one hand, or neutral factor accumulation on the other (aggregate income and households grow in strict proportion).

I will now present some results in terms of simulations. All of the results are intuitive, some are found formally in earlier papers and I am quite sure that all of the qualitative properties of all results have no dependence on the specific parameters or other assumptions used in these specific examples. The initial “calibration” point is as follows: at productivity one, the income and price elasticity are 1.333 and the share of $X$ in consumption is 0.5; the value of $\beta = 2/3$ is used in this example and throughout the paper. As productivity grows without bound the income and price elasticities approaches one and the $X$ consumption share approaches its marginal value of $2/3$ in Figure 1.

With the neutral and equal productivity growth in both sectors, the production frontier of the economy is growing radially, but demand is shifting toward good $X$. This generates a movement around the frontier toward $X$, so the relative price of $X$ rises as shown in Figure 2. But this generates the usual Stolper-Samuelson effect on relative factor prices, so the rental ($r$) - wage ($w$) ratio $r/w$ is rising as shown in Figure 2. Suppose we interpret capital as skilled labor or human capital and $L$ as unskilled labor. A neutral productivity growth generates an increase in the wage gap between skilled and unskilled labor. Thus we can get a wage gap (skill premium) phenomenon driven by the demand side of the general-equilibrium model without appealing to trade or to skill-biased technical change.

Now consider differences in relative endowments, beginning with the two countries identical, under the assumption of costless trade. Move capital from f to h and labor from h to f, implying the $Z$ rises in f and falls in h by an equal and opposite amount. Their Engels curves will move apart in Figure 1 but they remain parallel. Figure 3 shows the effect of widening the endowment differences. It graphs the share of world consumption and production in each country. The consumption shares in this exercise would be constant at 0.5 under homothetic demand. But under our assumption that the capital intensive good is the high-income-elasticity good, the consumption shares are positively correlated with their respective good’s production share.

Figure 3 gives a demand-side explanation for two phenomenon that have previously been identified and attributed to production-side causes. The positive correlation between production and consumption shares has been one (of several) definition of “home bias”. Secondly, the volume of trade is less under our assumptions than is predicted under a standard Heckscher-

---

4 An extension possibly relates to the Prebisch (1950) - Singer (1950) hypothesis. If the countries differ in relative endowments (standard Heckscher-Ohlin), then neutral productivity growth in both countries will lead to a terms-of-trade deterioration for the labor-abundant country: the “south”.
Ohlin model and thus offers a demand-side explanation for the empirical puzzle of “missing trade” in Trefler’s (1995) terminology. The amount of missing trade is identified in Figure 3 and note that it continues to grow in importance once countries are specialized: production specialization cannot continue to increase but consumption specialization can. As noted earlier, non-homotheticity as a cause of missing trade was noted theoretically by Markusen (1986) is empirically verified in Hunter (1991), Cassing and Nishioka (2012), and Caron, Fally, and Markusen (2012). Closely related points in the Ricardian context are found in Matsuyama’s (2000) theory and in Fieler’s (2011) theoretical and empirical paper.5

3. Imperfect competition, prices and markups

In this section, we add scale economies, imperfect competition, and free entry and exit of firms in the $X$ industry in a standard model of Cournot competition, continuing with the assumption that $X$ is a homogeneous good. $Y$ is produced with constant returns under perfect competition. We assume segmented markets simply because the results are more interesting and in line with Simonovska for example, so the model is similar to Venables (1985) or Markusen and Venables (1988), the latter contrasting segmented and integrated markets cases.

To keep matter manageable, we will concentrate on the case of per-capita income differences arising from productivity difference, but the qualitative results are identical to the case where countries differ in endowment ratios. Suppose that there is just a single factor of production, $L$, and distinguish between the household count and the “effective” or productivity-adjusted supply of labor in each of the two countries. One unit of $Y$ uses one unit of effective labor supply and $X$ uses $c$ units of effective labor for marginal costs and $F$ units for fixed costs. The two countries could have identical aggregate incomes but different per-capita incomes: one country can have higher productivity but proportionately lower household count.

Good $Y$ is used as numeraire. Revenue ($R$) for a Cournot firm in country $i$ and selling in country $j$ is given by the price in $j$ times quantity of the firm’s sales. Price is a function of all firms’ sales.

$$R_{ij} = p_j(X_j)X_{ij}, \quad X_j \text{ is total sales in market } j: \quad X_j = \sum_i X_{ij}$$

Assume zero trade costs and segmented markets, with $c$ and $F$ having the same value across countries. Then any firm that operates will sell in both markets and will sell the same in each market as any other firm regardless of that particular firm’s home country: $X_{ii} = X_{ji}, X_{ij} = X_{ji}$. $n$ will then denote the total number of firms from both countries active in equilibrium, where $n$ is

---

5Markusen (1986) and Fieler (2011) explicitly explain large “north-north” versus small “north-south” (and south-south in Fieler) trade volumes. An entirely different explanation is advance in Markusen and Wigle (1990): north-south and south-south trade is small because the south is poor and because the pattern of world protection discriminates heavily against north-south and south-south trade.
determined by the usual free-entry zero-profit conditions. This zero-profit condition for a firm from i is as follows, with a corresponding condition for a firm from j.

\[ p_iX_{ii} + p_jX_{ij} - c(X_{ii} + X_{ij}) - F = 0 \]  \hspace{1cm} \text{zero profits} \quad (9)

The result that gives the Cournot markup of a firm is fairly well known and I will not re-derive it here: the markup in j (defined on price \( p_j \)) is given by the firm’s market share divided by the price elasticity of demand. With zero trade costs and equal marginal costs, each firm in selling in a market has the same market share as any other firm regardless of home country. So the market share is always just \( 1/n \) and the markup is \( 1/(n \varepsilon_j) \).

\[ m_{rj} = m_{rij} = p_j \left[ 1 - \frac{1}{n \varepsilon_j} \right] = c \quad \text{or} \quad p_j = \left[ \frac{n \varepsilon_j}{n \varepsilon_j - 1} \right] c \]  \hspace{1cm} \text{Cournot pricing} \quad (10)

Market clearing in country j is given from earlier results, similarly for i.

\[ p_j n X_{ij} + p_j n X_{ji} = \beta (M_j - M_j^0) \quad M_j^0 \equiv \left( \frac{1 - \beta}{\beta} \right) p_z z L_j \]  \hspace{1cm} \text{market clears} \quad (11)

The elasticity of demand \( \varepsilon \) is given from (8) above. Also use (10) to replace \( p \) in \( m^0 \).

\[ \varepsilon_j = \frac{m_j}{m_j^0} \quad m_j^0 = p_j \frac{1 - \beta}{\beta} z = \frac{n \varepsilon_j}{n \varepsilon_j - 1} \frac{1 - \beta}{\beta} z c \]  \hspace{1cm} (12)

Using the second equation of (12), the first becomes

\[ \varepsilon_j = \frac{m_j}{m_j^0} \quad m_j^0 = \frac{n \varepsilon_j}{n \varepsilon_j - 1} \frac{1 - \beta}{\beta} z c \]  \hspace{1cm} (13)

which in turn reduces to

\[ \left[ \frac{n \varepsilon_j - 1}{n \varepsilon_j} \right] \left[ \frac{\varepsilon_j - 1}{\varepsilon_j} \right] = \frac{1 - \beta}{\beta} z c \]  \hspace{1cm} \varepsilon_j \text{ is decreasing in } m_j, n \text{ constant} \quad (14)

Recall that the number of firms \( n \) is common between i and j. We then get a “cross-section” result from (14): comparing two countries, the higher per-capita income country will have a lower price elasticity, higher markup and higher price level.

Add the zero-profit conditions for the representative firms in i and j in (9) together.
There are four Cournot pricing equations (10) for the two market supplies for the representative firm from each country. Multiply both sides of the four pricing equations (10) by the relevant outputs and set to zero (move c terms to the left-hand side). Add these four together and subtract them from (9). This will yield the condition that markups revenues equal fixed costs:

\[
\frac{p_j (X_{jj} + X_{ij})}{n \varepsilon_j} + \frac{p_i (X_{ii} + X_{ij})}{n \varepsilon_i} = 2F \quad \text{(markups revenues equal fixed costs)} \quad (15)
\]

Use the market-clearing equations in (11) to substitute for the revenue terms in (15).

\[
\frac{\beta (M_j - M_j^0)}{n^2 \varepsilon_j} + \frac{\beta (M_i - M_i^0)}{n^2 \varepsilon_i} = 2F \quad (16)
\]

and replace \( \varepsilon_i \) and \( \varepsilon_j \) with (12) \( \frac{m_i}{(M_i - M_i^0)} = \frac{M_j}{(M_j - M_j^0)} \). (16) becomes

\[
\frac{\beta (M_j - M_j^0)^2}{n^2 M_j} + \frac{\beta (M_i - M_i^0)^2}{n^2 M_i} = 2F \quad (17)
\]

Solving for \( n \), we have

\[
n = \left[ \left( \frac{(M_j - M_j^0)^2}{M_j} + \frac{(M_i - M_i^0)^2}{M_i} \right) \left( \frac{\beta}{2F} \right) \right]^{1/2} \quad \text{and} \quad (18)
\]

\[
n \varepsilon_j = \left[ \left( \frac{(M_j - M_j^0)^2}{M_j} + \frac{(M_i - M_i^0)^2}{M_i} \right) \left( \frac{\beta}{2F} \right) \right]^{1/2} \left[ \frac{M_j}{M_j - M_j^0} \right] \quad (19)
\]

Consider two identical countries with equal aggregate incomes \( M = M_i = M_j \)

\[
n \varepsilon = \left[ \left( \frac{2 (M - M^0)^2}{M} \right) \left( \frac{\beta}{2F} \right) \right]^{1/2} = \left( \frac{\beta}{F} \right)^{1/2} \left[ \frac{M}{M - M^0} \right] = \left( \frac{\beta M}{F} \right)^{1/2} \quad (20)
\]

which (when inverted) gives a simple formula for the common markup.

\[
\frac{1}{n \varepsilon} = \left[ \frac{F}{\beta M} \right]^{1/2} \quad \text{common markup for identical countries} \quad (21)
\]
The markup falls with a growth in aggregate income due to the pro-competitive effects of entry (Venables 1985, Markusen and Venables 1988). But the interesting thing about (25) is that non-homotheticity washes out. Holding aggregate income constant, increase per-capita income (increase productivity offset by fewer households). This lowers the price elasticity of demand but this is exactly offset in this special case by more entry. Recognizing that this last result is derived for identical economies only, we can thus suggest that non-homotheticity does not have a “time-series” effect on markups as per-capita income grows over time, but does show up in the “cross-section” comparison between countries with different per-capita incomes.

Results for this section are illustrated in Figure 4. The “cross-section” result is shown in the left-hand panel. The two-countries are identical at a value of 0.5 on the horizontal axis. Then productivity increases in h and falls in f holding aggregate incomes constant and equal (household numbers move inversely with productivity). With the price of $Y$ equalized between countries, this means that the price index is greater in country h as shown in Figure 4. The results on prices and markups is consistent with those in Wong (2003), Hummels and Lugovskyy (2009), and Simonovska (2009). Qualitatively, the same result occurs if we maintain equal productivities equal but transfer $K$ from f to h and $L$ from h to f.

A volume-of-trade result is shown in the right-hand panel of Figure 4 where the two countries are identical. Productivity is rising along the horizontal axis and absolute endowment lowered to maintain identical and constant aggregate incomes. The higher per-capita income moving to the right leads to a shift in consumption to $X$ and to an increase in intra-industry trade, inter-industry trade being zero. Thus trade volume increases relative to aggregate income. The same result will of course hold under monopolistic competition (Markusen 1986, Bergstrand 1989). A consequence is that gravity equations should show trade rising with per-capita income, aggregate income held constant, a topic discussed in the next section.

Figure 5 shows the results of an extension of this model to include horizontal multinational firms with two-factor economies, making the model identical to that of Markusen and Venables (1998). Trade costs are added to the model outlined above, and firms can invest in a foreign plant for an added fixed cost less that the initial fixed cost F. Both the fixed costs and marginal costs of $X$ production are capital intensive. In the simulation, the fixed costs of a two-plant horizontal multinational are 1.4 times that of a single-plant national firm and trade cost are 15 percent. The exercise shown in Figure 5 is similar to that in Figure 3, in that the countries relative endowments are shifted relative to the center of the horizontal axis of Figure 5 where the countries are identical (the horizontal axis is the NW-SE diagonal of the world Edgeworth box). The per-capita income of country h is higher to the left and that of country f is higher to the right. The vertical axis gives the share of multinational firms (headquartered in either country) as a share of all firms active in equilibrium.

As shown in Markusen and Venables, multinational arise when the countries are relatively similar. When their relative endowments differ significantly, production costs will lead production to be concentrated in national (single plant) firms located in the capital abundant country. Now make preferences homothetic, with a high income-elasticity of demand for the
capital-intensive X good as before. Now demand will be concentrated in the capital abundant country, which reinforces the production cost advantage of single-plant national firms located in the capital abundant country. Except where the countries are nearly identical in Figure 5, the share of horizontal multinationals will be less in case of non-homothetic demand for any difference in per-capita (relative endowment) differences. To the best of my knowledge, this has not been tested (since no one has generated the hypothesis), but should be very easy to do so by simply adding per-capita income difference to the standard regression equations such as those in Carr, Markusen and Maskus (2001).

4. Non-homotheticity and Trade Policy: an Application to Trade and the Environment

In this section, we consider an extension of the analysis to an international policy question that is of current interest: the relationship between trade policy and international environmental policy with a global pollutant such as CO₂. Assume that we have two final consumption goods (X, Y), one environmental good (E), and two countries (n, s), where n (north will) be the higher per-capita income country.

Final goods are produced by a CET (constant elasticity of transformation) technology, one input \( L \).

\[
Z = \left[ \delta X^\epsilon + (1 - \delta) Y^\epsilon \right]^{\frac{1}{\epsilon}} = \gamma L_z \quad \epsilon > 1 \quad \sigma = \frac{1}{\epsilon - 1}
\]

where \( \sigma \) is the elasticity of transformation and \( \gamma \) is a productivity parameter determining per-capita income. Allowing \( \delta \)’s to differ between \( n \) and \( s \) allows for comparative advantage and a basis for trading goods. Pollution is modeled as a reduction in the “endowment” of good \( E \) and is proportional to the total aggregate output of \( Z \) in both countries:

\[
\text{Pollution} = \text{Reduction in } E \text{ endowment} = (Z_n + Z_s) = \gamma (L_{zn} + L_{zs})
\]

Input \( L \) also used for an abatement activity \( A \). Aggregate labor in each country is then divided between production (\( Z \)) and abatement (\( A \): \( L = L_z + L_a \) where, as before, \( L \) is also the number of households.

\[
\text{Abatement} = \text{Addition to the } E \text{ endowment} = \gamma (L_{zn} + L_{zs})
\]

Abatement is financed by a consumption taxes in \( n \) and \( s \): \( ptax_n, ptax_s \). Thus public policy, via the consumption tax, can determine that allocation of the composite input \( L \) between production and abatement.

The advantage of this simple model is that it implies a “neutrality” in several senses that avoids important questions but not ones of interest here. By neutrality I mean:
no pollution-intensive sector
=> no comparative advantage in polluting sector
=> no factor-intensity, factor-endowment issues
=> no pollution-from-consumption-versus-production issue

The dominant model in the trade-and-environment literature has only one sector that pollutes, pollution is from production, not consumption. This leads to policy results that are very sensitive to:

=> which factor is intensive in which good
=> which good is the country’s comparative advantage good
=> whether pollution is from consumption or production

Here we avoid these issues.

Preferences are Stone-Geary, lower case letters for per-capita quantities

\[ u = x^\alpha y^\alpha (e + e_0)^\beta \quad 2\alpha + \beta = 1 \]  \hspace{1cm} (25)

where \( e_0 \) is an equal and exogenous initial endowment of environmental quality for each households in each country.

Let \( e^i \) denotes the demand for environment in country \( i \),
\( m^i \) is country \( i \)’s per-capita income
\( p_e^i \) is the price (willingness to pay) for environment in \( i \).

Consumer optimization yields:

\[ e^i = \max \left[ 0, (\beta - 1) e_0^i + \frac{\beta m^i}{p_e^i} \right] \]  \hspace{1cm} (26)

\[ e^i > 0 \quad \text{iff} \quad m^i > \frac{(1 - \beta)}{\beta} p_e^i e_0 \quad \iff \quad m^i > m^0 \]  \hspace{1cm} (27)

The result is analogous to that in the previous sections: up to the threshold per-capita income given in (27), there is no demand for additional (above the endowment level of) environmental quality. Once the threshold income is reached,

\[ e^i = (\beta - 1) e_0 + \frac{\beta m^i}{p_e^i} \]  \hspace{1cm} (28)
Because environment is a public good, efficiency equates the sum of the marginal willingness to pay over all consumers to marginal cost (Samuelson rule). To keep it simple here, normalize the populations of both countries to one, so that the sum of the prices over population just equals the price.

At a constant price, the income elasticity of demand for $e$ is greater than one. In equilibrium, $e$ must be the same for all consumers in all countries (perfect global public good). So for (27) to hold, it must be that the willingness to pay $p_e$ differs across countries. This willingness to pay can be found by simply inverting (28).

$$p_e^i = \frac{\beta m^i}{e + (1 - \beta)e_0}$$  \hspace{1cm} (29)

The (per capita) income elasticity of willingness to pay for a given $e$ is one.

Assume that $X$ and $Y$ are freely traded, so that their relative prices are the same in the two counties and indeed in the numerical results shown below these prices are always equal ($\delta_n = 1 - \delta_s$ so the countries are mirror images). We can then treat $X$ and $Y$ as a composite commodity $Z$, and let $Z$ be numeraire: it’s world price equals one. Equations (23) and (24) above imply that the ratio of marginal costs of an added unit of environment $E$ to the marginal cost of an extra unit of (composite commodity) $Z$ equals one.

Assume zero consumption taxes initially. Suppose we calibrate the model so that the demand price (willingness to pay) in country $s$, $P_e^s$, equals one initially in (29). Then the marginal rate of substitution between $E$ and $Z$ in country $s$ is equal to one, which is equal to the ratio of marginal costs. Country $s$ will have no incentive to use a consumption tax to transfer production from $Z$ to $E$. But if this is true for country $s$, then the marginal rate of substitution in the higher per-capita income country $n$ is greater than one since $p_e^n > p_e^s$ from (29). At zero taxes, we have

$$\frac{P_e^n}{P_e^s} = \frac{m_e^n}{m_e^s} = \frac{m_e^n}{mc_e^s} = 1$$  \hspace{1cm} (30)

It is optimal for country $n$ to transfer resources toward abatement and away from $z$ production. With the consumer price of $Z$ (and of $X$ and $Y$ individually) given by $p_z^n(1 + t^n)$, the environmental distortion from country $n$’s point of view is internalized with a tax such that:

$$\frac{P_e^n}{p_z^n(1 + t^n)} = \frac{m_e^n}{mc_e} = 1$$  \hspace{1cm} (30)

---

Because environment is a public good, efficiency equates the sum of the marginal willingness to pay over all consumers to marginal cost (Samuelson rule). To keep it simple here, normalize the populations of both countries to one, so that the sum of the prices over population just equals the price.
From (29), the optimal tax is given by

\[ 1 + t^n = \frac{p^n_e}{p^s_e} = \frac{m^n}{m^s} \quad \text{or} \quad t^n = \frac{m^n - m^s}{m^s} \quad (31) \]

Of course, the introduction of this tax will lead to an increase in \( e \) for both countries, lowering the willingness to pay for environment for country \( s \). But with its tax at zero initially, there is no change in its tax. There is an additional change due to the fact that country \( n \) “shrinks” as a trading partner for \( s \) (fewer resources are devoted to \( X \) and \( Y \) production). Finally, the tax in (31) is not jointly optimal of course, because it does not take into consideration the benefits to country \( s \) from the improved environmental quality. So now we turn to some numerical simulations to see just how this works in general equilibrium.

A popular idea in the trade and environment literature is a “Kuznets curve”, which postulates that, as a country’s per-capita income rises, environmental quality at first deteriorates and then eventually begins to improve. Figure 6 show the results of a simulation in which, for simplicity and clarity, I make the countries identical (or call it one country). The experiment is to increase productivity equally for both factors of production, raising household per-capita income. The horizontal axis is per capita income of both countries and the vertical axis measures per-capita welfare (left) and environmental quality (right) where both are normalized to one initially.

At a low level of per capita income, the inequality in (27) fails to hold, and it is not optimal to do any abatement. Thus over the initial range of per-capita income shown in Figure 6, an increase in productivity reduces environmental quality since \( Z \) increases while no abatement is undertaken. At a critical level of per-capita income, the inequality in (27) binds and it is optimal to begin abatement. Because environment has an income elasticity greater than one, the quality of the environment in a rich country eventually exceeds that in the very poor country. While both \( Z \) and \( E \) grow with the increased productivity, the share of the input \( L \) devoted to abatement grows relative to the share devoted to \( Z \).

Above, I alluded to the point that country \( n \)'s tax is jointly sub-optimal because it does not take into account its beneficial effect on the welfare of country \( s \). Figure 7 shows the results of a simulation which compares the non-cooperative and Nash-bargaining cooperative outcomes for homothetic versus non-homothetic demand. The model is calibrated such that, with zero or equal consumption taxes to finance abatement, the two countries have equal total incomes, but country \( n \) has a per-capita income 50 percent higher than country \( s \) (thus \( n \) has \( 2/3 \) the number of households relative to country \( s \)). The Nash-bargaining solutions use the non-cooperative taxes as the disagreement outcome.

The black boxes in Figure 7 give the outcomes with homothetic demand. As shown, Nash bargaining with equal bargaining weights raises the tax rates significantly, from 0.19 to 0.35. In the non-cooperative outcome, each country’s tax rate is ignoring 50 percent of the
benefit of it’s tax which are going to the other country, though in general equilibrium the optimal cooperative taxes are a little less than double the non-cooperative rates.

The red boxes in Figure 7 give the outcomes with non-homothetic demand. The results are likely intuitive and do not require much discussion. The difference in per-capita shifts the non-cooperative tax rate to a higher rate for country $n$ and a lower rate for country $s$. These form the disagreement values for Nash bargaining, and the Nash bargaining outcome leads to country $n$ less than doubling its tax and country $s$ more than doubling its tax. I think that the intuition is that a tax increase by $n$ does not benefit $s$ that much ($s$ has a low evaluation of the added environmental quality) while an increase in $s$’s tax has a much bigger benefit to $n$.

The experiment in Figure 8 makes a point that is perhaps poorly appreciated in the policy sphere. Suppose that countries agree to bargain over an “equal sharing rule” for CO2 emission levels, meaning they wish to set the same proportional reduction target for all countries. It seems to me that this is roughly the US position in the debate, and it is strongly resisted by poorer countries. The horizontal axis of Figure 8 graphs the equal taxation rates, while the vertical axis indexes the welfare levels of the two countries, normalize to equality initially (total incomes are the same at zero taxes as in Figure 7).

The upper curve in Figure 8 gives country $n$’s welfare and also that of country $s$ when their per-capita income levels are the same. Welfare of both countries is maximized by tax rates of 0.44. Then lower productivity and raise the number of households in country $s$ such that country $s$’s total income is the same at zero taxes. The middle curve in Figure 8 shows the welfare of country $s$ when it has a per-capita income level only one-fifth that of country $n$. In this case, the welfare of $s$ equals a maximum at a tax of 0.23 and, given the equal sharing rule, $s$ will not agree to any higher common tax rate.

The lowest curve in Figure 8 show the welfare of country $s$ when it’s per-capita income level is only 10 percent of country $n$’s level. In this case, equal sharing (tax) rule gives only minuscule gains to country $s$: it’s welfare is maximized at the common tax rate of only 0.04. A slightly larger difference and country $s$ cannot gain from any common positive tax rate. The point is that an equal tax or equal abatement target may be infeasible with a large difference in per-capita income or, more correctly, the outcome that leaves both country no worse off is zero taxes.

The final exercise in this section is to consider the gains from linking issues in international negotiations. There has been much discussion in political and policy circles about the appropriateness and wisdom of tying issues together in international bargaining. Most commonly, this is tying trade liberalization with either environment and/or labor standards commitments. Typically, the argument is over whether or not large, rich countries should tie reduced trade barriers for poor countries with the insistence that the latter should adopt tougher environmental and/or labor regulations on wages, working conditions, health and safety policies and so forth. Most of the debate, in my view, has been ideological, with a clear left-right split on this issue.
The model I have developed here is easily adapted to shed some light on this issue. Figure 9 considers a case where country $n$ has five times the total income and the per-capita income of country $s$ (so same number of households). This creates the classic optimal-tariff motive which is stronger for the large country $n$. But country $n$ will also want to have a higher tax and abatement effort due to its higher per-capita income.\footnote{Trade in goods is generated by comparative advantage due to differences in the $\delta$’s in (22). Country $n$ has a $\delta = 0.9$ and country $s$ has a $\delta = 0.1$.}

On the right-hand side of Figure 9, the results for the computation of “optimal” non-cooperative trade and abatement taxes is shown. Country $n$’s optimal tariff is four times as high as that for country $s$, and country $s$’s optimal abatement tax is zero. Figure 9 uses these values as a disagreement outcome and graphs the payoffs to country $n$ lowering its tariff in exchange for country $s$ raising it abatement tax. Country $n$’s abatement tax and country $s$’s tariff are held constant. The vertical axis shows the payoffs, relative to the non-cooperative disagreement outcome, for the exchange of a trade concession by $n$ for an abatement effort by $s$. The value on the right-hand side of Figure 9 gives the values for the maximum point of this payoff surface. Country $n$ lowers its tariff from 1.55 (155 percent) to 0.65, and country $s$ raises its abatement tax and effort from 0 to 0.16 at the maximum value of the Nash bargaining function.

The point of this exercise is that, when we leave ideology aside, there is a gain to linking issues to produce a Pareto superior outcome. The intuition behind this is that the large, rich country $n$ doesn’t really care much about trade liberalization: it has a high optimal tariff but it loses little from lowering it, but it cares a lot about abatement by country $s$. Country $s$ cares a great deal about country $n$’s tariff, but it does not cost it too much to make some abatement effort. This creates an arbitrage opportunity for both countries to exploit these differences.

5. Econometric Support

We have now made progress in estimating and testing the crucial assumption that produces some of the theoretical results above: the positive relationship between skilled-labor and capital intensity of a good in production and the income elasticity of demand for that good in consumption (Caron, Fally and Markusen, 2012). We use the GTAP7 data set (94 countries, 57 sectors, 5 factors, 4700 observations) which gives us the relevant capital, skilled, and unskilled-labor plus land and resources used in value added and also use an input-output structure to add the indirect use of $K$ and skilled/unskilled $L$ from intermediate use.

For the empirical work, we use the “CRIE” (constant relative income elasticity) utility function that has been recently used successfully by Fieler (2011). One problem with the data is that it is expenditure data, not separate price and quantity data, and we are concerned that trade costs may be important; in particular, trade costs could give a correlation between a country’s specialization in consumption and specialization in production, leading to home bias and missing
trade. Thus we conduct a two-stage estimation, first estimating gravity equations to give price indexes for each country. Second, we then estimate the parameters of the demand equations, controlling for prices from the first stage, via a constrained, non-linear least squares. The constraints are the income balance or adding-up conditions for each country, so that the fitted values of consumption for each country satisfy adding up.

The results of this empirical analysis are as follows. (1) deviations of income elasticities of demand from one (homotheticity) are large and highly statistically significant. (2) the positive correlation between a good’s income elasticity of demand and its skilled-labor intensity in production is large, in excess of 50 percent, and highly statistically significant. The result is true with and without trade costs, though controlling for trade costs lowers the correlation a little. (3) the estimation shows that non-homotheticity can explain about 1/3 of missing trade when the latter is calculated using the assumption of identical and homothetic preferences everywhere. (4) the estimation helps explain why rich countries trade more with other rich countries, and why rich countries trade more as a share of GDP. (5) using the estimated demand system to recalibrate the GTAP data, a counter-factual simulation of the world model (about 144,000 equations and unknowns) show that an uniform, Hicks-neutral productivity improvement in all sectors in all countries leads to an increase in the skilled-wage premium in all countries, as demand shifts toward the high-income-elasticity, skilled-labor-intensive goods.

Thus the crucial assumption underlying the trade theory work here (the income-elasticity, skill-intensity correlation) receives strong support, as do a number of predictions of the model.

6. Summary

As suggested in the introduction, there are bits-and-pieces of theoretical and empirical analysis about the role or roles for per-capita income in determining trade flows. But there is little unity and by and large per-capita income is not given much of a place as an important determinant of trade. This paper tries to unify and connect the bits, and to offer further ideas about how per-capita income might matter. I offer a “generic” model that I hope might prove useful for graduate teaching, a sort of all-in-one model that nests a number of other contributions.

The model imposes a variant of Stone-Geary preferences (used before by a number of authors) on top of a traditional 2x2x2 Heckscher-Ohlin model. Maintained hypotheses are that labor endowments in the HO model are proportional to the number of households and that the skill/capital-intensive good in the HO model is the high income-elasticity-of-demand good. The latter assumption is testable and falsifiable. Results from the model offer a demand-side explanation for a range of phenomena including (a) home bias in consumption, (b) the mystery of the missing trade, (c) a growing skill premium in an environment of growing productivity and (d) the differing effects of growth depending on whether it is through growth in per-capita or simply total income.

I then add an assumption of increasing returns to scale in the capital-intensive, high-
income-elasticity industry with free entry and exit of firms, Cournot pricing and segmented markets: a common framework in the so-called new trade theory and strategic trade-policy literatures. This generates some interesting and testable results, in particular higher markups and higher price levels in higher per-capita-income countries, and more trade between higher per-capita-income countries, aggregate income held constant. Horizontal multinational activity will be more concentrated among countries with similar per-capita income in addition to similar levels of total income. As in the case of the competitive examples, some of the implications have already received good empirical support.

A final section considers how non-homothetic preferences can inform trade policy and, in particular, the analysis of trade and the environment. Modeling environmental quality as a high income-elasticity good affected by a global pollutant, it is straightforward to illustrate several policy implications such as a (a) non-monotonic Kuznets curve, (b) the effect of per-capita income on cooperative and non-cooperative abatement policies by two nations, (c) a problem with policy proposals for equal abatement efforts across countries, and (d) a potentially valuable role for linking issues in international negotiations such as trade and the environment or trade and labor standards.

I conclude by noting that a crucial assumption of the model, the link between income elasticities of demand and factor intensities in production is receiving strong empirical support as are several implications of the model.

REFERENCES

Directly Relevant: focusing on Heckscher-Ohlin, oligopoly pricing, non-homothetic preferences across sectors (homogeneous goods within sectors)


Bernasconi, Claudia (2009), “New Evidence for the Linder Hypothesis and the two Extensive Margins of Trade”, working paper, University of Zurich.


Chenery, Hollis and Moshe Syrquin (1975), Patterns of Development 1950-70, Oxford University Press.


*Related, largely focusing on product quality and/or intra-country income inequality (with regrets, not analyzed here)*


Figure 1: Growth through productivity versus neutral factor accumulation (growth in the number of households)

Engels curve: neutral factor accumulation (number of households)

Engels curve: productivity growth (household per-capita income)

Figure 2: Skill premium from the demand side (identical countries)

Productivity level: countries identical

Both lines flat with homothetic demand
Figure 3: Home bias and the mystery of the missing trade

Correlation between production / consumption shares = 0.20

Figure 4a: Markups and price level: productivity differences (aggregate income held constant)

Proportional markup difference: country h to f
Proportional price index difference: country h to f
All values = 0 with homothetic demand
Figure 4b: Volume of trade related to per-capita income

Volme of trade relative to aggregate income

All values = 1 with homothetic demand

Per-capita income level, countries identical

Figure 5: Multinational entry depends on total and per-capita income

Share of world labor in h (capital share = 1 - labor share)
Figure 6: Welfare, environmental quality

Per-capita welfare level

Environmental quality index

Per-capita income level

Figure 7: Effect of differing per capita income on equilibrium

Cooperative Nash bargaining:
- h higher per-capita
- Nash eq. default option

Non-cooperative Nash equilibrium:
- h higher per-capita

h's optimal tax at f's tax = 0
- h higher per-capita

f's optimal tax at h's tax = 0
- equal per-capita
Figure 8: Infeasibility of an equal tax bargaining rule ($p_{taxn} = p_{taxs}$) when countries differ greatly in per-capita income: $PCI_N = 10*PCI_S$

Country n's welfare

Country s's welfare: $PCI_N = 5*PCI_S$

Country s's welfare: $PCI_N = 10*PCI_S$

Equal taxes: $p_{taxn} = p_{taxs}$ (countries sizes identical)

Figure 9: Gains from cooperative linked bargaining. Lower tarh traded for higher ptaxf

Nash eq:
- $tarh$: 1.55
- $ptaxh$: 0.39
- $tarf$: 0.20
- $ptaxf$: 0.00

Nash bargain:
- $tarh$: 0.65
- $ptaxh$: 0.39
- $tarf$: 0.20
- $ptaxf$: 0.16