University of Colorado - Department of Economics
Econ 7818 - Introduction to Probability and Asymptotic Theory - Fall 2024
Professor Carlos Brunet Martins-Filho

Office. Economics Building 105

Meetings. Tuesdays and Thursdays 9:30 AM - 10:45 AM at ECON 119.

Starting on the second week of this course there will be in-person weekly recitations conducted by a Teaching Assistant. The time and location for these recitations will be announced by the end of the first week of classes. Attendance is expected and strongly encouraged.

Office hours. Office hours will be held Tuesdays and Thursdays from 4:30 PM - 5:30 PM. If you need an appointment outside these hours send an email to carlos.martins@colorado.edu and I will try to accommodate your request.

Class URL. http://spot.colorado.edu/∼martinsc/7818.html.

Prerequisites. Successful completion of ECON Math Camp or consent of instructor.

Objectives. This is the first course of a first year two-course Ph.D. sequence in Econometrics. The course objectives are:

- to introduce you to fundamental tools and concepts from probability and asymptotic theory needed for a rigorous study of the limiting behavior of estimators and test statistics that emerge form the study of statistical/econometric models
- if time permits, to introduce you to the classical linear regression model and accompanying estimators and test statistics

Grades. Your course grade depends on your performance on a midterm and a final examination. Relevant dates and points are given below.

Evaluation	Points	Date
Midterm examination	40	October 17
Final examination	60	December 14, 1:30 PM - 4:00 PM

Sets of homework questions, together with their answers, will be provided to help you prepare for the exams. Being able to answer these questions is an important indicator that you are learning.

Support material and reference books.

Class notes are available for this course. They will be available as PDFs on the class website. Study them carefully. In addition, the following books have very good presentations of some of the material we will cover.

A. Mathematics, Probability and Asymptotic Theory

1. Apostol, T., 1974, Mathematical Analysis, Addison Wesley, New York.
2. Bartle, R., 1966, Elements of Integration, John Wiley and Sons, New York.
3. Davidson, J., 1994, Stochastic Limit Theory, Oxford University Press, Oxford.

4. Dhrymes, P., 1989, Topics in Advanced Econometrics: Probability Foundations, Springer Verlag, New York.

5. Grimmett, G.R. and D.R. Stirzaker, 1992, Probability and Random Processes, Oxford University Press, Oxford.

6. Jacod, J. and P. Protter, 2000, Probability Essentials, Springer, Berlin.

7. Resnick, S. I., 2005, A Probability Path, Birkhauser, Boston.

B. Econometrics

1. Amemiya, T., 1985, Advanced Econometrics, Harvard University Press, Cambridge, MA.

2. Davidson, J., 2000, Econometric Theory, Blackwell Publishers, Oxford, UK.

3. Newey, W. and McFadden, D., 1994, Large sample estimation and hypothesis testing. In Handbook of Econometrics IV, R. Engle and D. McFadden Editors, Chapter 36.

Topics.

1. Probability spaces
   (a) σ-algebras
   (b) Measure: properties and characterizations
   (c) Null sets and complete measure spaces
   (d) Probability measure
   (e) Conditional probability (on events) and independence of events
2. Construction of probability measures
   (a) Carathéodory's Extension Theorem
   (b) Lebesgue measure on $(\mathbb{R}^n, \mathcal{B}(\mathbb{R}^n))$
   (c) Distribution functions and probability measures on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$
3. Measurable functions
   (a) Random variables, vectors and elements
   (b) Transfer of measures between measure spaces
   (c) σ-algebras generated by random elements
4. Integration
   (a) Abstract integration and expectation of random elements
   (b) Beppo-Levi Theorem
   (c) Lebesgue's monotone and dominated convergence theorems
   (d) $\mathcal{L}^p$ spaces, Hölder's and Minkowski-Riez's inequalities
   (e) Riemann, improper Riemann and abstract integrals
5. Independence of random variables
   (a) Independence of σ-algebras

(b) Finite dimensional distribution functions
(c) Product measures
(d) Fubini's and Tonelli's Theorems
(e) Independence of random variables

6. Convergence of random elements
   (a) Almost sure convergence
   (b) Convergence almost everywhere and in probability
   (c) The Borel-Cantelli Lemma
   (d) Uniform integrability
   (e) Convergence in $\mathcal{L}_p$, Riez-Fisher Theorem
   (f) Convergence in distribution
7. Law of large numbers
   (a) Tail equivalence of sequences of random variables
   (b) A general law of large numbers (LLN) for independent sequences
   (c) Kolmogorov's and Markov's LLN
8. Conditional expectation
   (a) Hilbert spaces
   (b) The Projection Theorem
   (c) Conditional expectation and conditional probability (on σ-algebras)
9. Central limit theorems
   (a) Fourier transforms and its inverse (characteristic functions)
   (b) Lévy's, Feller's and Liapunov's central limit theorems
10. Linear regression models
    (a) Identification
    (b) Loss functions and Extremum (M) estimation
        i. Least squares (LS)
        ii. Maximum likelihood (ML)
        iii. Method of moments (MM)
    (c) Consistency and limiting distributions: LS, ML, MM
    (d) Asymptotic Efficiency

Important information.

- Classroom behavior

  Students and faculty are responsible for maintaining an appropriate learning environment in all instructional settings, whether in person, remote, or online. Failure to adhere to such behavioral standards may be subject to discipline. Professional courtesy and sensitivity are especially important with respect to individuals and topics dealing with race, color, national origin, sex, pregnancy, age, disability, creed, religion, sexual orientation, gender identity, gender expression, veteran status, marital status, political affiliation, or political philosophy. For more information, see the policies at www.colorado.edu/policies/classbehavior.html and at www.colorado.edu/studentaffairs/judicialaffairs/code.html#student_code.

- Accommodation for Disabilities, Temporary Medical Conditions, and Medical Isolation

  If you qualify for accommodations because of a disability, please submit your accommodation letter from Disability Services to your faculty member in a timely manner so that your needs can be addressed. Disability Services determines accommodations based on documented disabilities in the academic environment. Information on requesting accommodations is located on the Disability Services website. Contact Disability Services at 303-492-8671 or DSinfo@colorado.edu for further assistance. If you have a temporary medical condition, see Temporary Medical Conditions on the Disability Services website. If you have a temporary illness, injury or required medical isolation for which you require adjustment, please send me an e-mail.

- Preferred student names and pronouns

  CU Boulder recognizes that students' legal information doesn't always align with how they identify. Students may update their preferred names and pronouns via the student portal; those preferred names and pronouns are listed on instructors' class rosters. In the absence of such updates, the name that appears on the class roster is the student's legal name.

- Honor code

  All students enrolled in a University of Colorado Boulder course are responsible for knowing and adhering to the Honor Code. Violations of the Honor Code may include but are not limited to: plagiarism (including use of paper writing services or technology [such as essay bots]), cheating, fabrication, lying, bribery, threat, unauthorized access to academic materials, clicker fraud, submitting the same or similar work in more than one course without permission from all course instructors involved, and aiding academic dishonesty. Understanding the course's syllabus is a vital part in adhering to the Honor Code.

  All incidents of academic misconduct will be reported to Student Conduct & Conflict Resolution: StudentConduct@colorado.edu. Students found responsible for violating the Honor Code will be assigned resolution outcomes from the Student Conduct & Conflict Resolution as well as be subject to academic sanctions from the faculty member. Visit Honor Code for more information on the academic integrity policy.

- Sexual misconduct, discrimination, harassment and/or related retaliation

  CU Boulder is committed to fostering an inclusive and welcoming learning, working, and living environment. University policy prohibits protected-class discrimination and harassment, sexual misconduct (harassment, exploitation, and assault), intimate partner abuse (dating or domestic violence), stalking, and related retaliation by or against members of our community on- and off-campus. The Office of Institutional Equity and Compliance (OIEC) addresses these concerns, and individuals who have been subjected to misconduct can contact OIEC at 303-492-2127 or email CUreport@colorado.edu. Information about university policies, reporting options, and support resources including confidential services can be found on the OIEC website. Please know that faculty and graduate instructors must

inform OIEC when they are made aware of incidents related to these policies regardless of when or where something occurred. This is to ensure that individuals impacted receive outreach from OIEC about resolution options and support resources. To learn more about reporting and support for a variety of concerns, visit the Don?t Ignore It page.

- Religious Accommodations

  Campus policy requires faculty to provide reasonable accommodations for students who, because of religious obligations, have conflicts with scheduled exams, assignments or required attendance. Please communicate the need for a religious accommodation in a timely manner via e-mail.

  See the campus policy regarding religious observances for full details.

  www.colorado.edu/policies/fac_relig.html.