Homework 6

ECON 4818

Professor Martins

1. From your textbook answer questions 5, 8 from Appendix D in your textbook.

2. Let

$$A = \begin{pmatrix} 1 & 5 & 6 \\ 3 & 7 & 9 \\ 4 & 3 & 9 \\ 10 & 4 & 6 \end{pmatrix} \text{ and } B = \begin{pmatrix} -1 & 2 \\ 4 & -2 \\ -9 & 4 \end{pmatrix}.$$

Calculate the product AB. Is it possible to calculate BA? Why or why not?

3. Consider the matrix

$$X = \begin{pmatrix} 1 & X_{12} & X_{13} & \cdots & X_{1K} \\ 1 & X_{22} & X_{23} & \cdots & X_{2K} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{n2} & X_{n3} & \cdots & X_{nK} \end{pmatrix}.$$

What are the dimensions of the matrix X. Obtain the transpose of X, i.e., X^T. Write down the elements of the matrix $X^T X$.

4. Consider the following regression model:

$$Y_i = \beta_0 + X_{i1}\beta_1 + X_{i2}\beta_2 + X_{i3}\beta_3 + U_i \text{ where } i = 1, \cdots, n.$$

(a) Write this regression model in matrix format, i.e., $Y = X\beta + U$. What are Y, X, β and U and give their dimensions.

(b) Let $S_n(\beta) = (Y - X\beta)^T(Y - X\beta)$. What is $\frac{\partial}{\partial \beta}S_n(\beta)$?

(c) If $\hat{\beta}$ satisfies $\frac{\partial}{\partial \beta}S_n(\hat{\beta}) = 0$, what is $\hat{\beta}$ as a function of X and Y? What assumption is needed on $X^T X$ to obtain $\hat{\beta}$?

(d) If $\{U_i\}_{i=1}^n$ is a sequence of independent and identically distributed random variables with $E(U_i) = 0$, $E(U_i^2) = \sigma^2$ and $U^T = \begin{pmatrix} U_1 & U_2 & \cdots & U_n \end{pmatrix}$, what are $E(U)$ and $E(UU^T)$?