Homework 9
ECON 4818
Professor Martins

1. Consider the following regression model

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + U_i$$

where $i = 1, \cdots, n$, $E(U_i|X_{i1}, X_{i2}) = 0$, $V(U_i|X_{i1}, X_{i2}) = \sigma^2$, $U_i \sim N(0, \sigma^2)$ for all i and $\{U_i\}_{i=1,\cdots,n}$ forms an independent sequence of random variables.

1. Suppose you estimate $E(Y|x_1, x_2) := \beta_0 + \beta_1 x_1 + \beta_2 x_2$ using $\widehat{E}(Y|x_1, x_2) = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2$, where $\hat{\beta}_j$ for $j = 0, 1, 2$ are least squares estimators. Is $\widehat{E}(Y|x_1, x_2)$ an unbiased estimator of $E(Y|x_1, x_2)$?Prove.
2. Obtain the variance of $\widehat{E}(Y|x_1, x_2)$.
3. Construct a α-confidence interval for $E(Y|x_1, x_2)$.

2. Consider the following linear regression model

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + U_i$$

where $i = 1, \cdots, n$ and $U_i \sim N(0, \sigma^2)$ for all i and $\{U_i\}_{i=1,\cdots,n}$ forms an independent sequence of random variables. In class we have shown that if we want to test that $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ against the alternative hypothesis that some $\beta_j \neq 0$ for some $j = 1, 2, 3$ we can do so by using the statistic

$$f = \frac{m^{-1}(R\hat{\beta} - r)'(R(X'X)^{-1}R')^{-1}(R\hat{\beta} - r)}{\hat{\sigma}^2} \sim F_{m,n-4}$$

where $R = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, $m = 3$, $r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and $\hat{\sigma}^2 = \frac{1}{n-4}\hat{U}'\hat{U}$ (refer to your notes for the definition of X, $\hat{U}$). Provide all assumptions needed to obtain that under the null hypothesis H_0 we have that $f \sim F_{m,n-4}$. For each assumption explain why and where it is needed in the proof of the distribution of f. Use mathematical arguments.