ECON 4818 - Econometrics
Professor Carlos Martins
Midterm 1
Date: 09.23.2025

INSTRUCTIONS: Your answers should be written on just one side of a sheet of paper. Use a new sheet of paper to start the answer of each question.

1. Question: Suppose X is a random variable such that $X \sim N(\mu, \sigma^2)$. Let $Y = aX+b$ where $a > 0$, and $b \in \mathbb{R}$ are non-random constants.

   a) (2 point) Obtain $E(Y)$ and $V(Y)$.

   b) (3 point) Given that Y is normally distributed, what is $P(Y \leq 1)$ as a function of a probability associated with a standard normally distributed random variable?

2. Question: Suppose X is a random variable with density given by

$$f(x) = \begin{cases} \frac{x^2}{9} & \text{if } 0 < x < 3 \\ 0 & \text{otherwise} \end{cases} .$$

   a) (1 point) Show that f is in fact a density.

   b) (2 points) Obtain $E(X)$ and $V(X)$.

3. Question:

   a) (2 points) Show that $V(X+Y) = V(X)+V(Y)+2\text{Cov}(X,Y)$. What is $V(X+Y)$ if X and Y are independent?

   b) (2 points) Show that if X and Y are random variables and $E(Y|X) = 0$, then $E(Y) = 0$.

   c) (4 points) If $E(Y|X) = a + bX$, where $a \in \mathbb{R}$ and $b > 0$ are non-random constants, can you obtain $E(X|Y)$? Explain using mathematical arguments.

4. Question: Let $\{Y_i\}_{i=1}^n$ be a collection of independent and identically distributed random variables. Assume that $E(Y_i) = \mu$ and $V(Y_i) = \sigma^2$ for all i.

   a) (2 points) Let $\hat{\mu} = \frac{1}{n}\sum_{i=1}^n Y_i$ be an estimator for μ. Obtain $E(\hat{\mu})$ and $V(\hat{\mu})$.

   b) (2 points) Suppose $\tilde{\mu} = \frac{1}{2}(Y_1 + Y_n)$ is another estimator for μ. Obtain $E(\tilde{\mu})$ and $V(\tilde{\mu})$.

   c) (2 points) Which of these estimators you prefer? Explain using mathematical arguments.