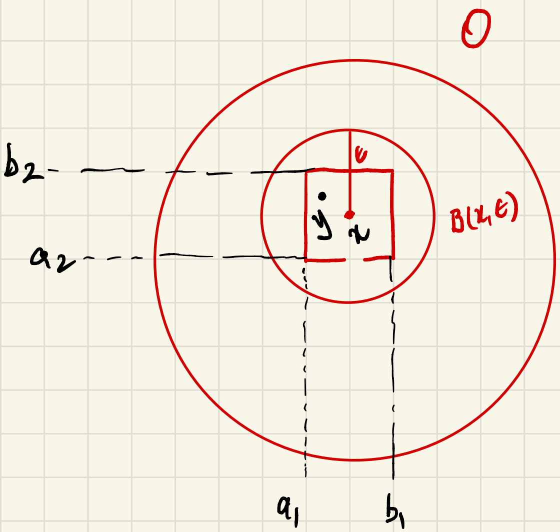
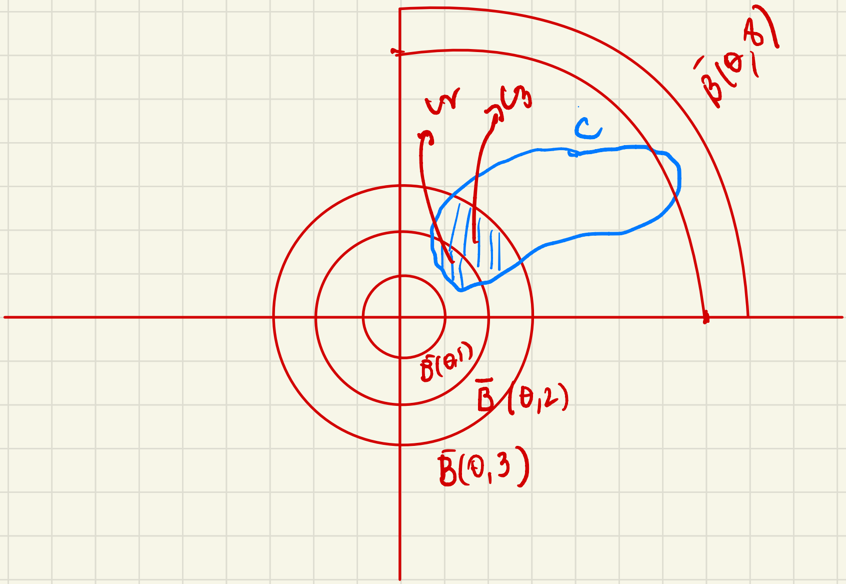
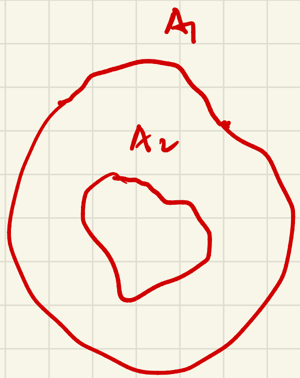


O
ε
$B(x, \varepsilon)$
y
x
b_2
a_2
a_1
b_1


U_2
U_3
C
$\bar{B}(0,8)$
$\bar{B}(0,1)$
$\bar{B}(0,2)$
$\bar{B}(0,3)$

1.

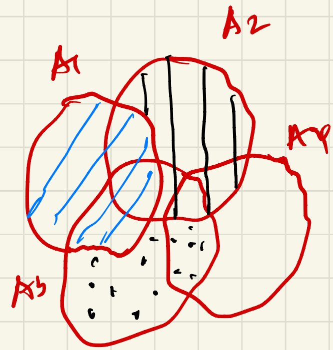


$A_1 = A_2 \cup (A_1 - A_2)$

$\mu(A_1) = \mu(A_2) + \mu(A_1 - A_2)$

$\Rightarrow \mu(A_1) \geq \mu(A_2)$

3.



$B_1 = A_1$

$B_2 = A_2 - A_1$

$B_3 = A_3 - \bigcup_{i=1}^{2} A_i$

2.

$A_1 \cup A_2 = A_2 \cup (A_1 - A_2)$

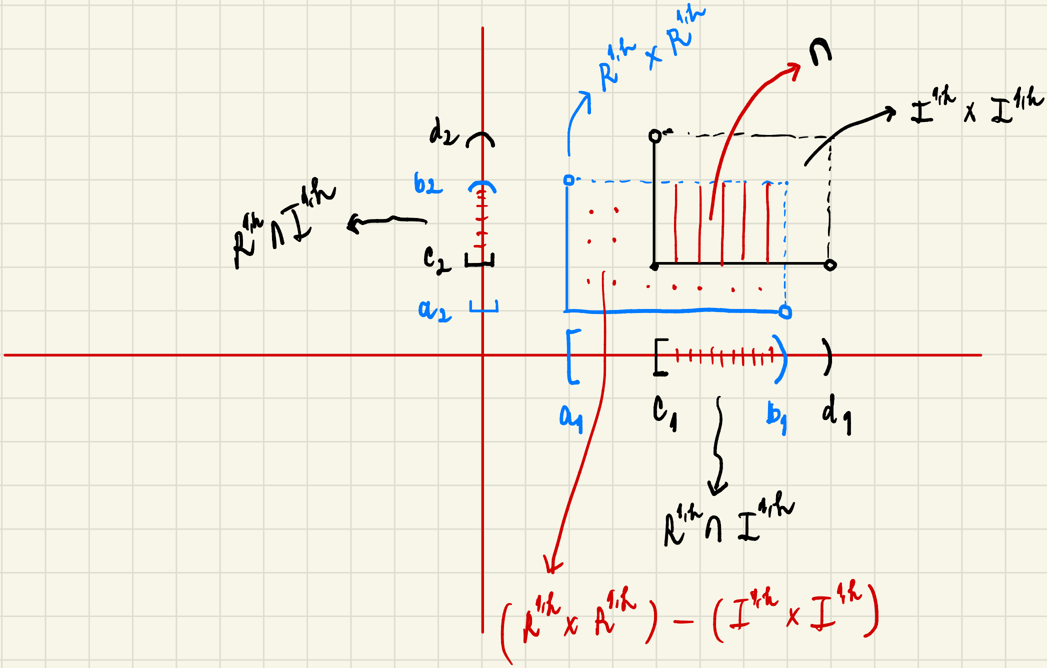
$A_1 = (A_1 \cap A_2) \cup (A_1 - A_2)$

$\mu(A_1 \cup A_2) = \mu(A_2) + \mu(A_1 - A_2)$

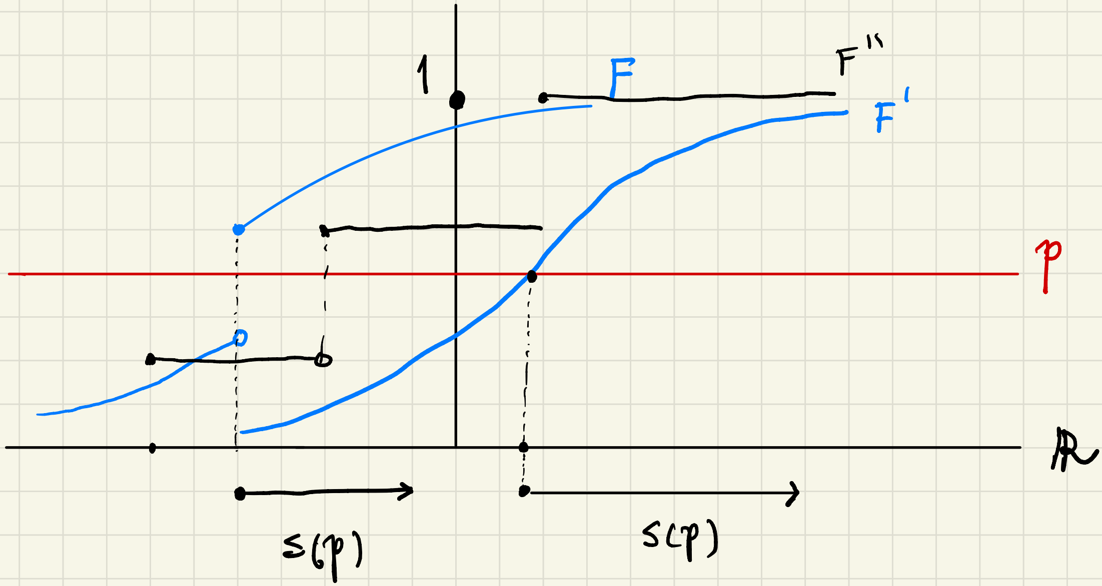
$\mu(A_1) = \mu(A_1 \cap A_2) + \mu(A_1 - A_2)$

$\mu(A_1 \cup A_2) - \mu(A_2) = \mu(A_1) - \mu(A_1 \cap A_2)$

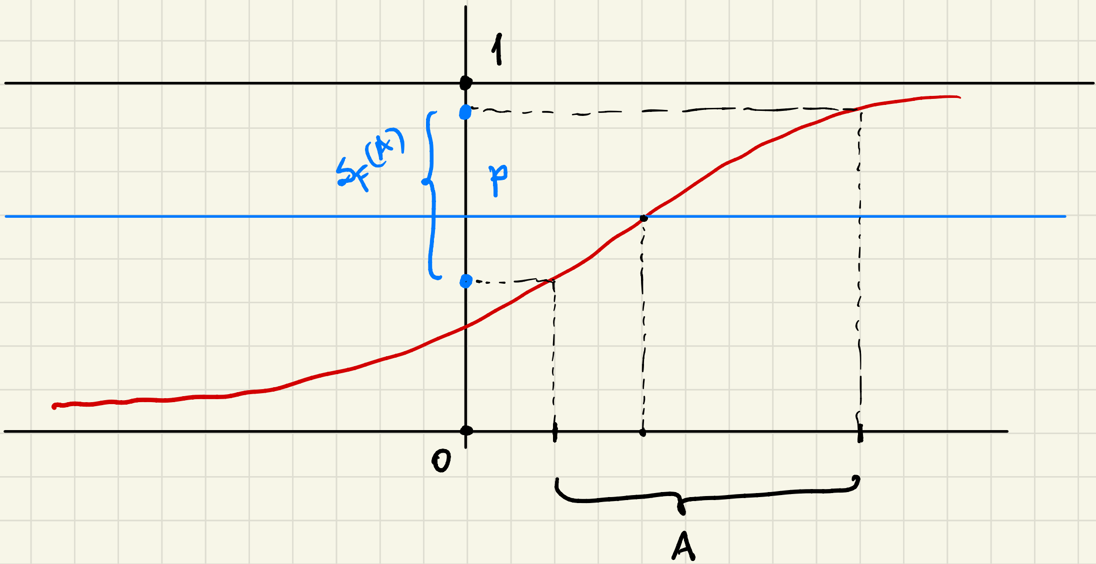
15.1



$R^{1,h} \times R^{1,h}$
$\cap$
$I^{1,h} \times I^{1,h}$
d_2
b_2
c_2
a_2
$R^{1,h} \cap I^{1,h}$
a_1
c_1
b_1
d_1
$R^{1,h} \cap I^{1,h}$
$(R^{1,h} \times R^{1,h}) - (I^{1,h} \times I^{1,h})$



47.1



1
$S_F(A)$
p
0
A

Let $X : (\Omega, \mathcal{F}, P) \longrightarrow (\mathbb{R}, \mathcal{B})$ be a random variable.

Then, $X^{-1}(B) \in \mathcal{F} \quad \forall \quad B \in \mathcal{B}$

$P(X^{-1}(B)) = P_X(B)$ where

$P_X : \mathcal{B} \longrightarrow [0,1]$. For $x \in \mathbb{R}$

$$P_X((-\infty, x]) = P_X(\{X \leq x\}) = P_X(X \in (-\infty, x])$$

$$:= F_X(x)$$

- $x < y \implies F_X(x) \leq F_X(y)$
- Let $x_n \downarrow x$. Then $(-\infty, x_n] \downarrow (-\infty, x]$

$$F_X(x) = P_X((-\infty, x]) = P_X\left(\lim_{n\to\infty} (-\infty, x_n]\right)$$

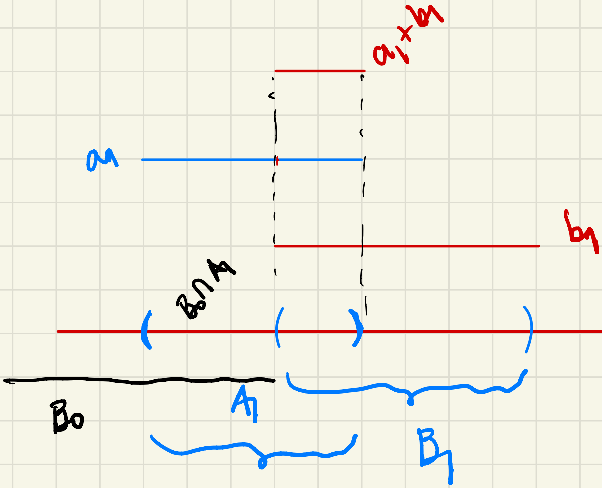
$$= \lim P_X(-\infty, x_n]$$

$$= \lim F_X(x_n)$$

- $$\lim_{x_n \uparrow \infty} F(x_n) = \lim_{x_n \uparrow \infty} P_X((-\infty, x_n])$$

$$= P_X\left(\lim_{x_n \uparrow \infty} (-\infty, x_n]\right) = P_X\left(\bigcup_{n \in \mathbb{N}} (-\infty, x_n]\right)$$

$$= P_X(\mathbb{R}) = 1$$


a1×b1
a1
b1
B0∩A1
B0
A1
B1

$E_{0,n}$: $0 \leq f(\omega) < 1/2^n$, $k=0$

$E_{1,n}$ $\frac{1}{2^n} \leq f(\omega) < 2/2^n$, $k=1$

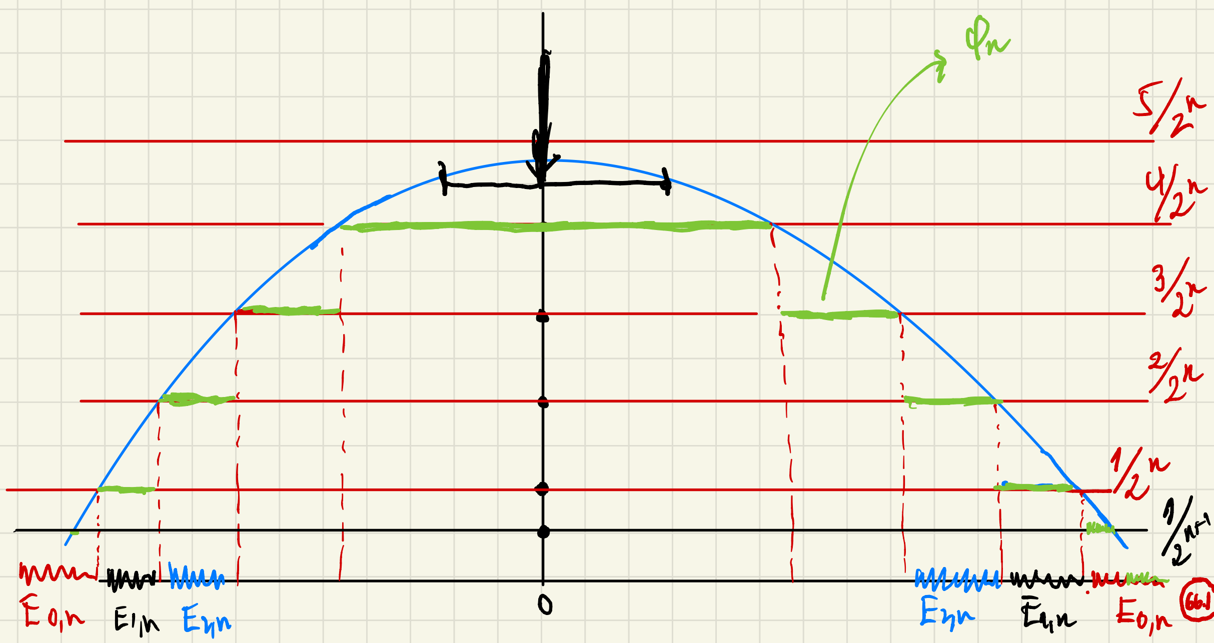
$E_{2,n}$ $\frac{2}{2^n} \leq f(\omega) < 3/2^n$, $k=2$

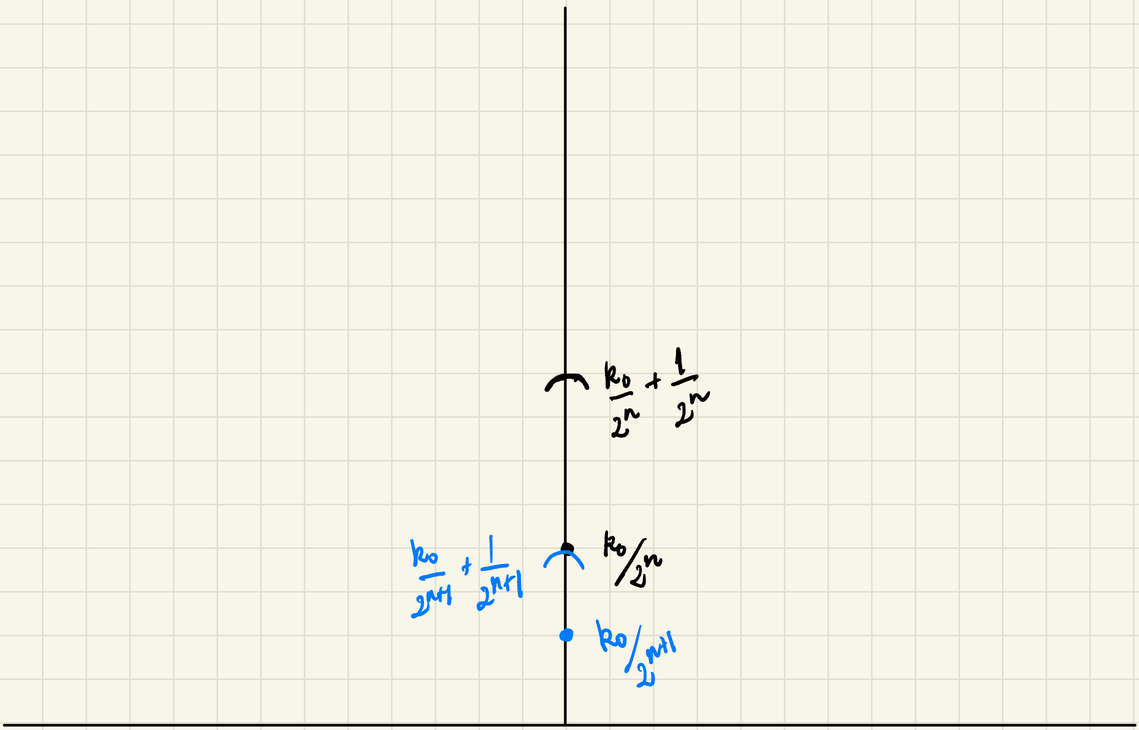
$\vdots$ $\vdots$

$E_{n2^n-1,n}$ $\frac{n2^n-1}{2^n} \leq f(\omega) < n$, $k=n2^n-1$

$k=n2^n$

$E_{n2^n} = f(\omega) \geq n$





$$\frac{k_0}{2^n} = \frac{2k_0}{2^{n+1}} = \frac{k}{2^{n+1}}, \quad k = 2k_0$$