Suppose $I = \{1, 2, 3, \ldots, n\}$

$$S = \{1, 2, 3\}$$

$$I_1 = \{1, 2\}$$

$$I_2 = \{3, 4\}$$

$$I_3 = \{5, 6, 7, \ldots, n\}$$

$$\mathcal{F}_{I_1} = \sigma\left(\bigcup_{i \in I_1} \mathcal{F}_i\right) = \sigma(\mathcal{F}_1 \cup \mathcal{F}_2)$$

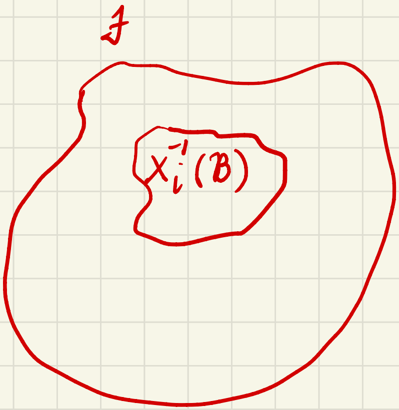
$$\mathcal{F}_{I_2} = \sigma\left(\bigcup_{i \in I_2} \mathcal{F}_i\right) = \sigma(\mathcal{F}_3 \cup \mathcal{F}_4)$$

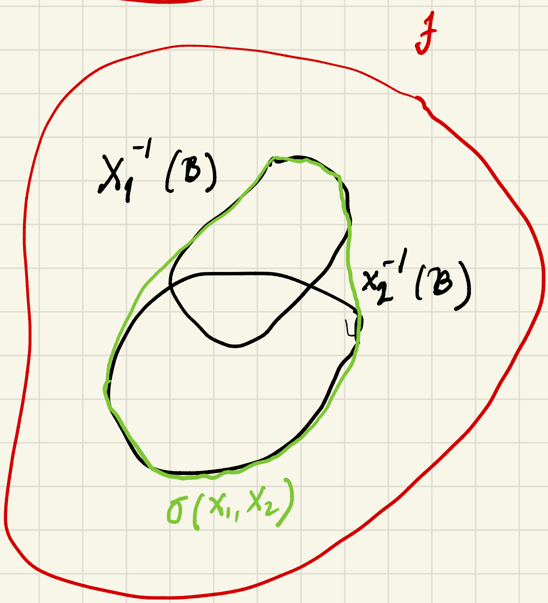
$$\mathcal{F}_{I_3} = \sigma\left(\bigcup_{i \in I_3} \mathcal{F}_i\right) = \sigma(\mathcal{F}_5 \cup \mathcal{F}_6 \cup \cdots \cup \mathcal{F}_n)$$

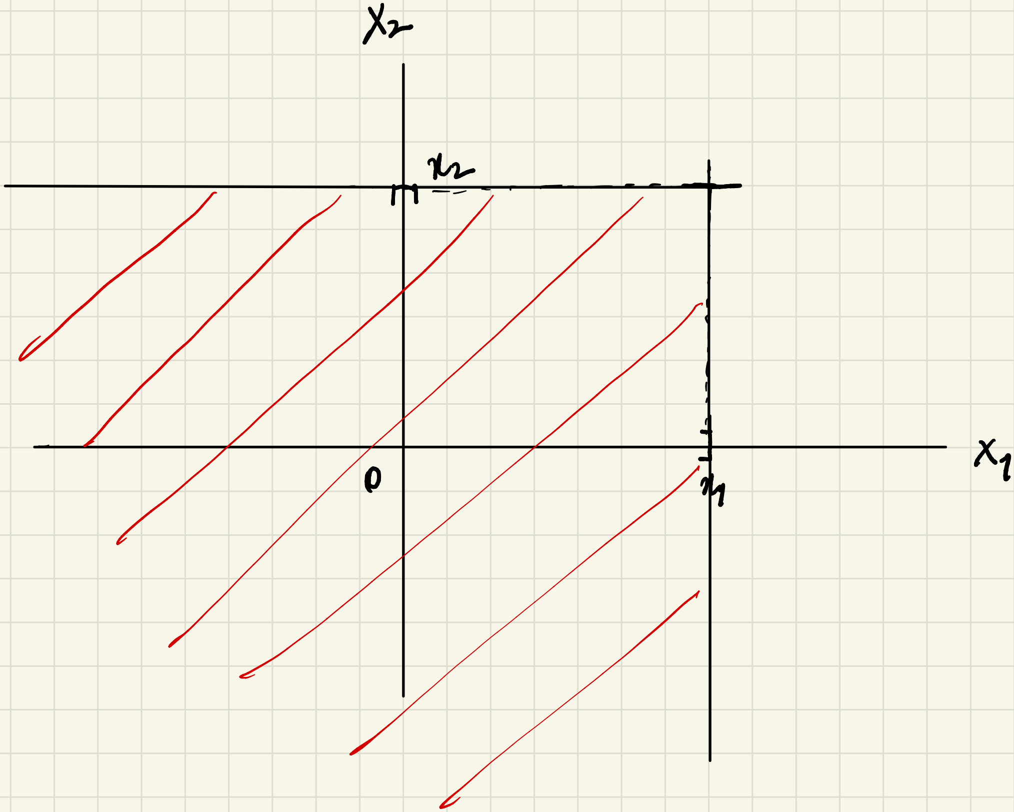
$\{\mathcal{F}_{I_s}\}_{s = 1, 2, 3}$ is independent.

- Any $T = \{t\}$ $\quad \mathcal{C}_{I_1} = \{ B_t : B_t \in \mathcal{F}_t \}$

$$X_i : (\Omega, \mathcal{F}, P) \longrightarrow (\mathbb{R}, \mathcal{B})$$

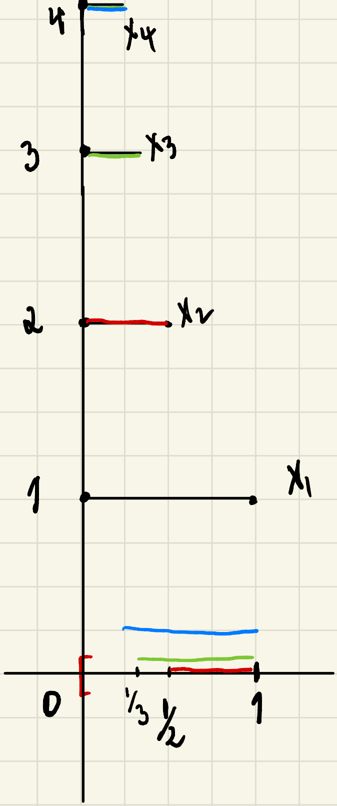






$$F_{\{1,2\}}(x_1, x_2) = P(X_1 \le x_1,\ X_2 \le x_2)$$

$$= P(\{\omega : X_1(\omega) \le x_1\} \cap \{\omega : X_2(\omega) \le x_2\})$$



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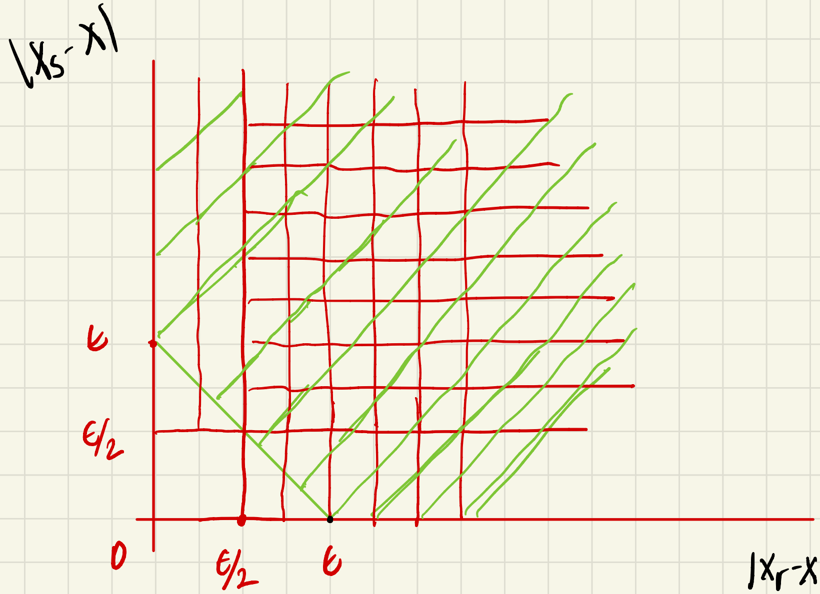

$$|x_s - x| = \epsilon - |x_r - x|$$

Suppose $\lim_{n\to\infty} \sum_{i=1}^{n} a_i = a < \infty$.

Then, $\forall \epsilon > 0 \ \exists \ N(\epsilon) \ni \forall n \geq N(\epsilon)$

$$\Big| \underbrace{\sum_{i=1}^{n}}_{S_n} a_i - a \Big| < \epsilon$$

$$\sum_{i=1}^{n+p} a_i = \sum_{i=1}^{n} a_i + \sum_{i=n+1}^{n+p} a_i$$

$$S_{n+p} = S_n + \sum_{i=n+1}^{n+p} a_i$$

$$\Rightarrow \quad S_{n+p} - S_n = \sum_{i=n+1}^{n+p} a_i$$

$\left| S_{n+p} - S_n \right| \leq \left| \sum_{i=n+1}^{n+p} a_i \right|$. In particular, for $p=1$ $\left| S_{n+1} - S_n \right| \leq |a_{n+1}| \downarrow 0$ as $n \to \infty$.

If S_n converges, it is Cauchy, hence as $n, p \to \infty$

$$\left| S_{n+p} - S_n \right| < \epsilon$$

- $\sum_{n=1}^{\infty} x_n$ is called a series

- $\left\{ \sum_{m=1}^{n} x_m ,\ n \in \mathbb{N} \right\}$ are called the partial series of the series

- The series converges if

$$\lim_{n\to\infty} \sum_{m=1}^{n} x_m < \infty$$

- A series is said to converge absolutely if

$$\lim_{n\to\infty} \sum_{m=1}^{n} |x_m| < \infty$$

<u>Theorem</u>: If a series converges absolutely, it converges.