

Thanks to Michael Tooley for the following proof.

$$(1) \Pr(A \& B \& C) = \Pr(A, (B \& C)) \times \Pr(B \& C) = \Pr(A \& B \& C) = \Pr(B, (A \& C)) \times \Pr(A \& C)$$

Therefore,

$$(2) \Pr(A, (B \& C)) = \frac{\Pr(B, (A \& C)) \times \Pr(A \& C)}{\Pr(B \& C)}$$

Replacing A, B, and C by H2, O, and B, we have

$$(3) \Pr(H2, (O \& B)) = \frac{\Pr(O, (H2 \& B)) \times \Pr(H2 \& B)}{\Pr(O \& B)}$$

Replacing A, B, and C by H1, O, and B, we have

$$(4) \Pr(H1, (O \& B)) = \frac{\Pr(O, (H1 \& B)) \times \Pr(H1 \& B)}{\Pr(O \& B)}$$

Dividing (3) by (4) gives one

$$(5) \frac{\Pr(H2, (O \& B))}{\Pr(H1, (O \& B))} = \frac{\Pr(O, (H2 \& B)) \times \Pr(H2 \& B)}{\Pr(O, (H1 \& B)) \times \Pr(H1 \& B)}$$

However

$$(6) \Pr(H2 \& B) = \frac{\Pr(H2/B)}{\Pr(B)} \text{ and}$$

$$(7) \Pr(H1 \& B) = \frac{\Pr(H1/B)}{\Pr(B)}$$

Substituting (6) and (7) into (5) then gives one

$$(8) \frac{\Pr(H2, (O \& B))}{\Pr(H1, (O \& B))} = \frac{\Pr(O, (H2 \& B)) \times \Pr(H2/B)}{\Pr(O, (H1 \& B)) \times \Pr(H1/B)}$$

Then, since we are given that

$$(9) \Pr(O, (H2 \& B)) > \Pr(O, (H1 \& B))$$

and

$$(10) \Pr(H2/B) \geq \Pr(H1/B)$$

It follows from (8) that

$$(11) \Pr(H2, (O \& B)) > \Pr(H1, (O \& B))$$