Linear-response reflection-coefficient of the recorder air-jet amplifier

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Recorder geometry

1. duct or windway
2. block
3. lip
4. upper chamfer
5. lower chamfer

\[ \frac{W}{h} \approx 4 \]

\[ Re = \frac{hU_0}{\nu} \approx 500 - 2500 \]

\[ \Rightarrow \text{Jet is laminar} \]

\[ \Rightarrow \text{Simplest flute-drive system} \]

Cross-section photo from Philippe Bolton
Reflectometer method

\[ S = \frac{\varphi_{out}}{\varphi_{in}} \]

Instrument assembled \( S_h S_p = 1 \rightarrow \)

- Pipe-tone oscillation condition: \(|S_h|^2 > 1.\)

Connect head to an absorbing termination \( \rightarrow \)

- Edge-tone oscillation condition: \( S_h \rightarrow \infty.\)
Apparatus I

Yamaha YRT-304BII tenor head

Controls mean pipe flow

vacuum pump

compressed air tank

V2

Recorder Head

Microphone Section

Attenuator

Driver

University of Colorado Boulder
Apparatus II
Calibration cell
$|S_h|$ at zero mean pipe flow
Phase of $S_h$ at zero mean pipe flow
Linear model

Verge, Hirschberg, Causse, JASA (1997)
Fabre, Hirschberg, Acustica (2000)

\[ q_j = J(\omega)(\alpha_p q + \alpha_j q_j) \]

\[ Z_h = \frac{p}{q} = r + i\omega M_1 + i\omega M_d \left( \frac{1 + (\alpha_p - \alpha_j)J(\omega)}{1 - \alpha_j J(\omega)} \right) + i\omega M_2 \]

\[ S_h = \frac{Z_h - Z_0}{Z_h + Z_0} \]

follow Verge
\[
\begin{aligned}
\alpha_p &= 2/\pi \\
\alpha_j &= 0.38 \\
M_d &= 0.88\rho/H \\
Z_0 &= \rho c/(\pi R^2)
\end{aligned}
\]

\[ M \text{ is fit at } J = 0 \quad M = M_1 + M_d + M_2 \]
Jet model

Jet gain parameter $g$

$$J(\omega) = -g \frac{U_0}{W} \frac{1}{i\omega} \left( 1 - e^{\mu \tilde{W}} e^{-i\omega \tilde{W}/c_p} \right)$$

$$U(y) = U_0 sech^2(y/b)$$ Bickley jet profile with $b = \frac{2}{5} h$ (follow Verge)

$$\tilde{W} = W + d$$ Jet path length includes chamfers
Jet model $c_p$ and $\mu$  

Mattingly and Criminale, Phys. Fluids (1971)
Jet transfer function $J(\omega)$ (for $g = 1$)

$\mathcal{S}_t = f \frac{\hat{W}}{U_0}$

$|S_h|^2 > 1$

$|S_h|^2 < 1$

if not oscillating

Increasing $f$
$|S_h|$ at zero mean pipe flow

fit parameters: jet gain $g = 0.145$

total inertance $M$
Phase of $S_h$ at zero mean pipe flow

fit parameters: jet gain $g = 0.145$

total inertia $M$
$|S_h|=1$ points at zero mean pipe flow

$St = \frac{f\widetilde{W}}{U_0}$

| $|S_h|^2 > 1$ |
| $|S_h|^2 < 1$ |
| $|S_h|^2 > 1$ |

$\approx 0.21\lambda$
Normal playing region is bounded by measured $|S_h|=1$

- $7\lambda/4$
- $5\lambda/4$
- $3\lambda/4$

- $A=440$ Hz pitch
- 10 cent intervals

Diagram showing frequency vs. blowing pressure with data points indicating intervals and pressure levels.
$|S_h|$ versus mean pipe flow

- **80 Pa, $Q_p/Q_j = 0.000$**
  - Blocked flow
  - $g = 0.145$

- **80 Pa, $Q_p/Q_j = 0.069$**
  - Edge-tone threshold
  - $g = 0.200$

- **80 Pa, $Q_p/Q_j = 0.465$**
  - Edge-tone oscillation

- **80 Pa, $Q_p/Q_j = 1.000$**
  - $\approx$ same as blocked
Conclusions

- Linear-response reflection measurements on an unmodified recorder head are possible for mean pipe flow $\approx 0$ and for mean pipe flow $\approx$ jet flow.

- Mean pipe flow can be used to control the jet gain $g$.

- With an absorbing termination, the head shows edge-tone oscillations for intermediate values of mean flow. The oscillation frequency is very close to the frequency of the gain peak. Edge-tone oscillations disappear at high incident amplitudes.

- The linear model after Verge shows fair agreement with the data. A jet deflection model gives $g = 0.58$ but data fits $g = 0.145$. Jet phase velocity $c_p$ is too large at higher St. Effects of chamfers? Segoufin JASA (2004), Giordano JASA (2014).

- The observed $|S_h| = 1$ boundary at $St = 0.27$ agrees closely with the low-blowing-pressure boundary for pipe-tone oscillations under normal playing conditions.
Supplemental Slides
Questions

• Can the linear model be refined for linear-response conditions? (Is the jet gain really too large? Why are the higher $|S_h| = 1$ boundaries at wrong $St$? Can parameter estimates be improved?)

• Can this measurement method be used to characterize gain-saturation? Could we then infer a useful model of the head that could help explain the observed limit cycles?

• Can the linear model be extended to describe pipe-tone and edge-tone saturation behavior?
Some questions in recorder acoustics

**Dynamics**
Can we model periodic limit cycles vs blowing pressure and fingering?
Can we understand multiphonic limit cycles?
Can we understand why some notes “sound” more easily?

**Structure**
What is the role of chamfers?
What is the role of windway length and curvature?
What is the role of lip asymmetry?
Why do some instruments “burble”?
Why do fingering patterns of recorders differ from those of traversos?

**Modeling**
How well do existing lumped models work?
Can gain saturation be understood using lumped models?
What are useful observables for computational experiments?
Kinds of experiments

Complete instrument
Radiated steady sound field vs. blowing pressure, geometry amplifier saturated
Internal steady sound field vs. blowing pressure, geometry amplifier saturated
Flow visualization amplifier saturated
Transients briefly unsaturated

Instrument in parts
Free jets no amplifier
Embouchure impedance (transverse flute) no amplifier
Unblown normal modes no amplifier
Tone hole properties, tone hole arrays no amplifier
Central jet velocity $U_0$

Assume Poiseuille profile at the duct exit.

→ must measure jet flow
Reflectometer method I

\[ S = \frac{\varphi_{out}}{\varphi_{in}} \]

\[ Z = \frac{p}{q} \]

1. When the instrument is assembled \( S_h S_p = 1 \) or \( Z_h + Z_p = 0 \).
2. If the system is linear, solutions for real \( \omega \) give oscillation thresholds.
3. The pipe is passive and almost lossless: \( |S_p|^2 \ll 1 \). This implies:

Pipe-tone oscillation condition: \( |S_h|^2 > 1 \).
Reflectometer method II

1. Measure $S_h$ with a matched transmission line and signal source. A microphone array on the line is used to infer $\phi_{in}$ and $\phi_{out}$. The experimenter can control the wave amplitude, frequency and the mean pipe flow.

2. With an absorbing termination, the system only oscillates when $S_h \to \infty$

Edge-tone oscillation condition: $S_h \to \infty$.

Coltman JASA (1968): impedance head & tuner, transverse flute, in saturation
Thwaites & Fletcher JASA (1983): “slotted line” SWR method, flue pipe, linear response