Linear-response reflection-coefficient of the recorder air-jet amplifier

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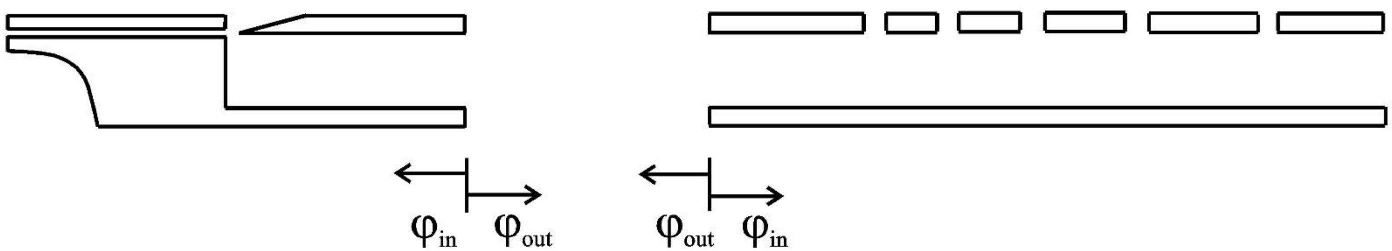
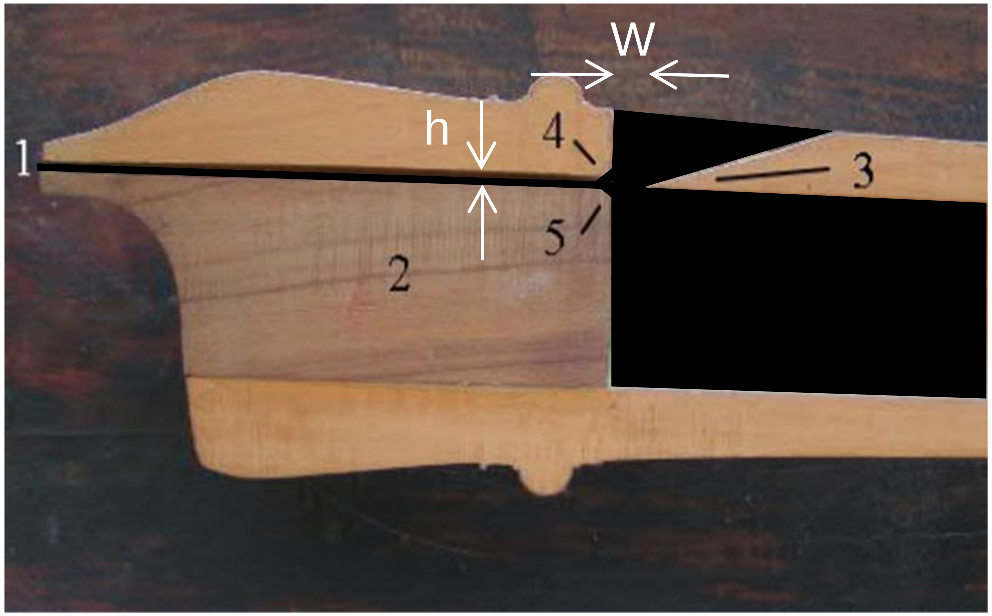


Recorder geometry



1. duct or windway
2. block
3. lip
4. upper chamfer
5. lower chamfer

$$W/h \approx 4$$

$$Re = \frac{hU_0}{\nu} \approx 500 - 2500$$

→ Jet is laminar

- Fixed duct
- Laminar jet
- We try to measure linear response

→ Simplest flute-drive system

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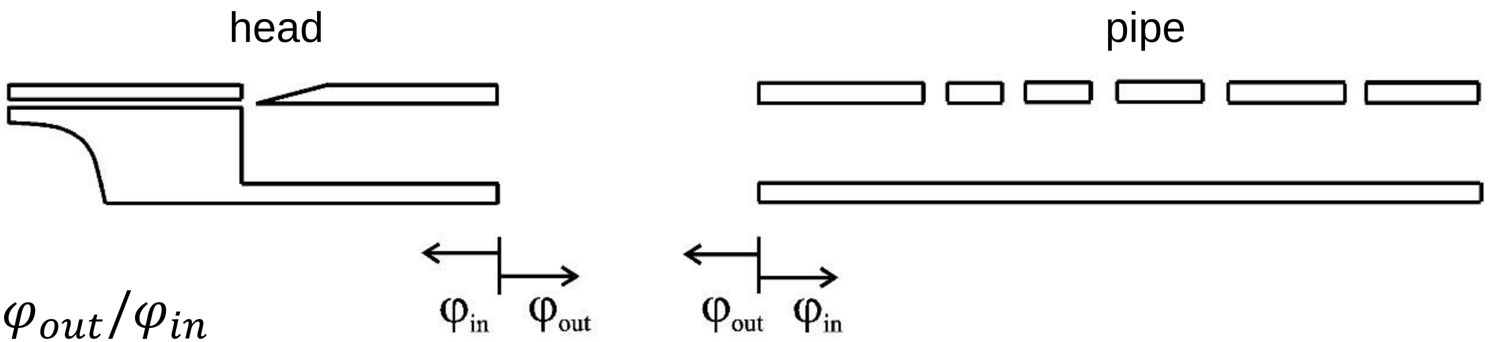
Cross-section photo from Philippe Bolton

Reflectometer method



$$S = \varphi_{out}/\varphi_{in}$$

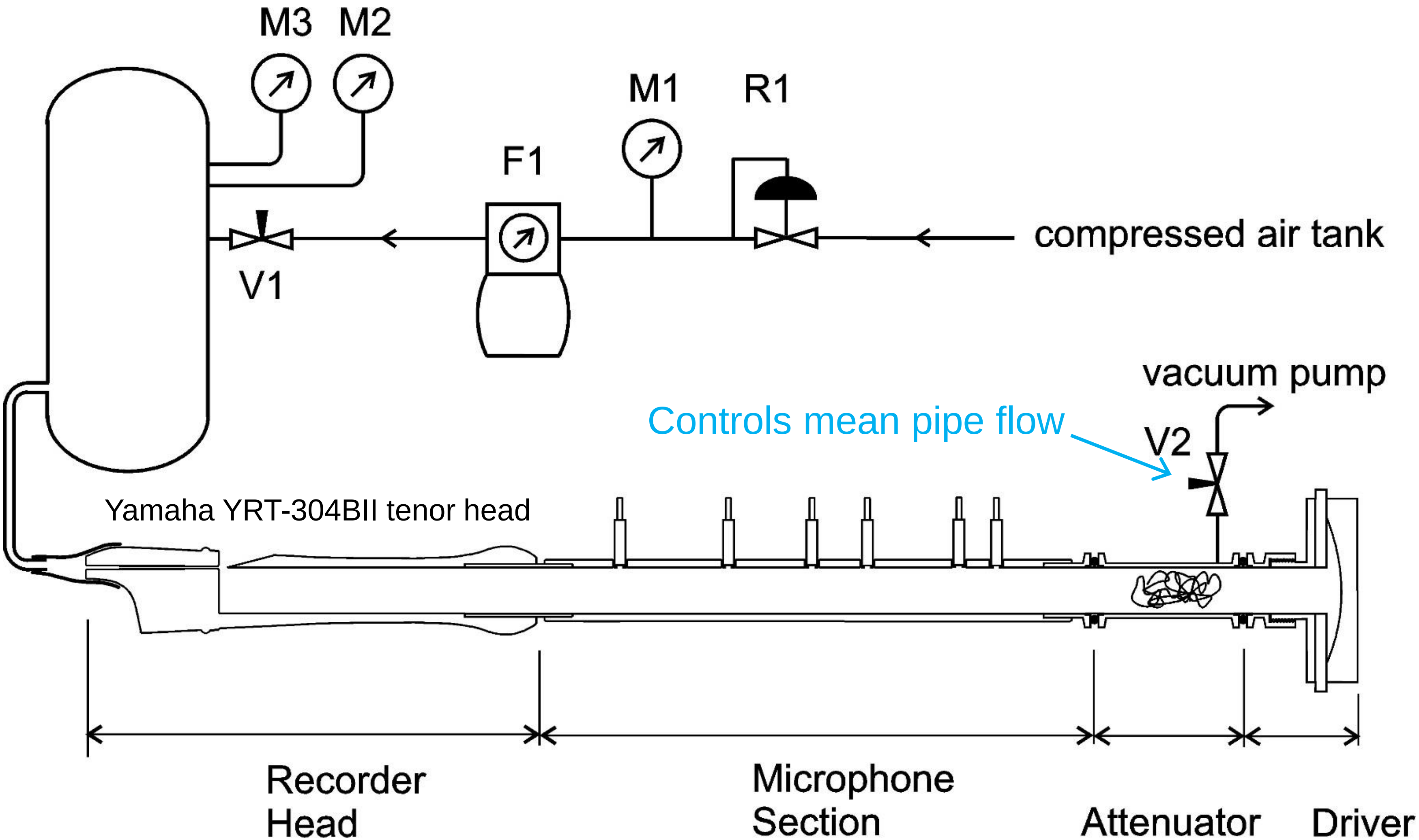
Instrument assembled $S_h S_p = 1$ →

Pipe-tone oscillation condition: $|S_h|^2 > 1$.

Connect head to an absorbing termination →

Edge-tone oscillation condition: $S_h \to \infty$.

Apparatus I



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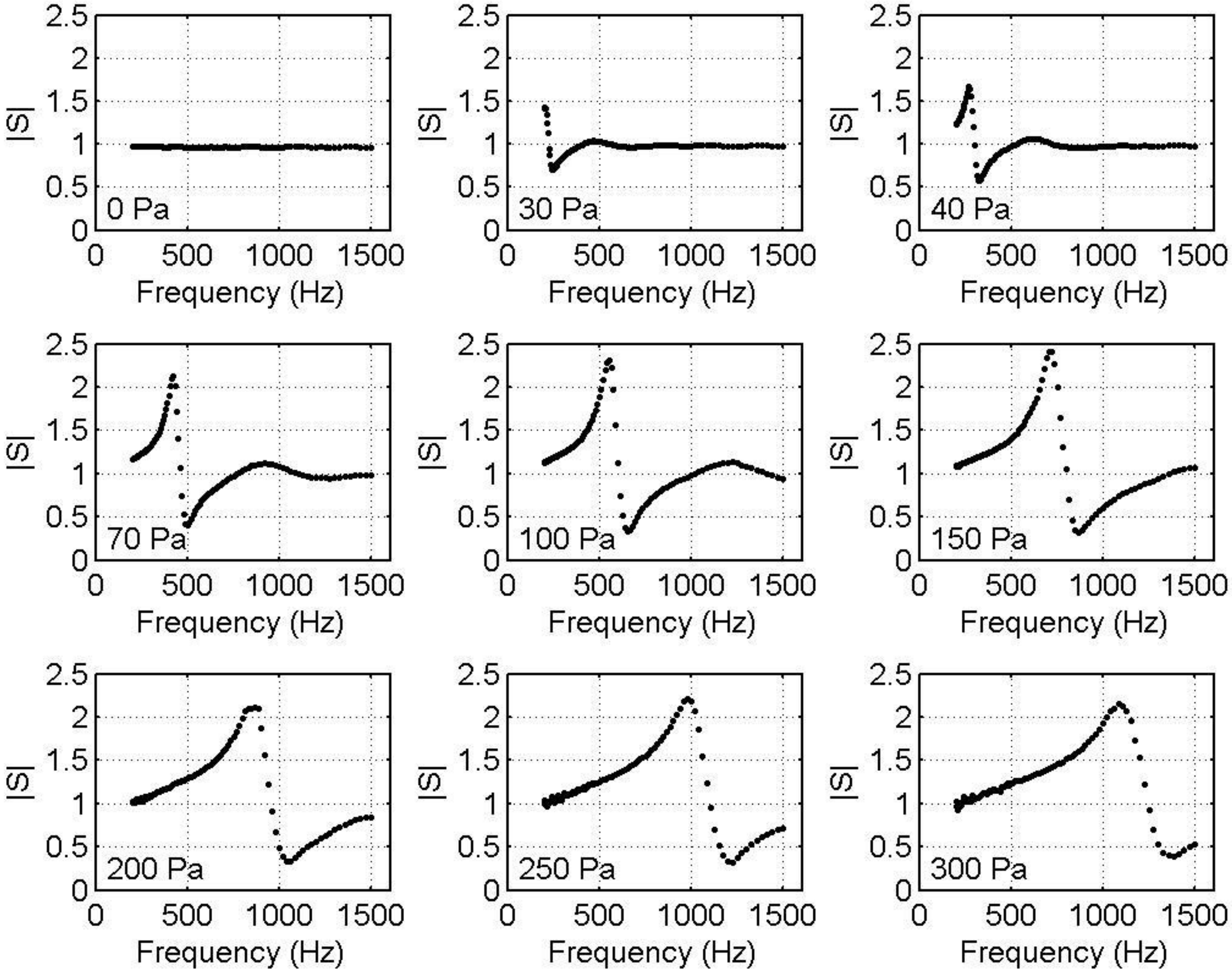
Apparatus II

Calibration cell

$|S_h|$ at zero mean pipe flow



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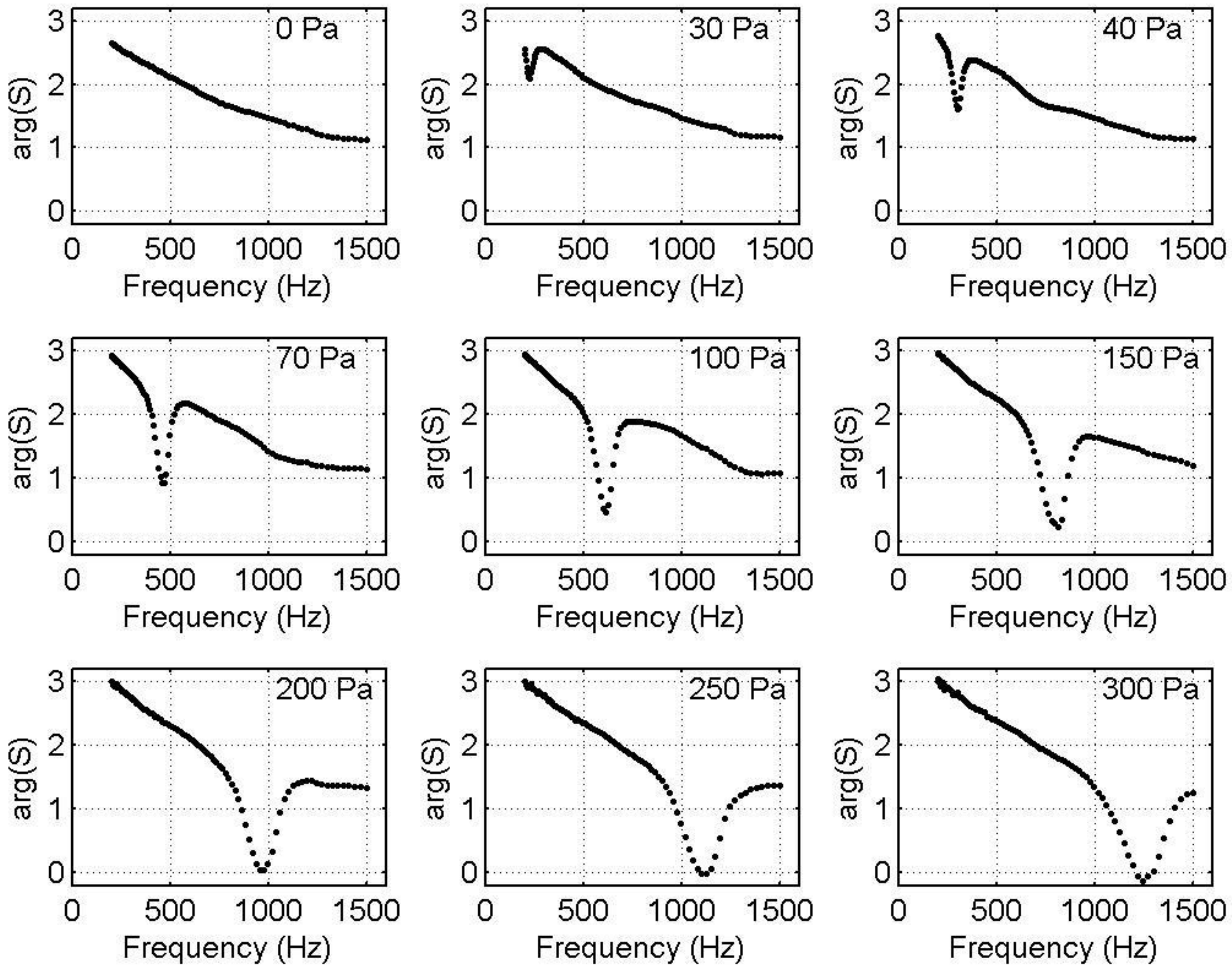
Phase of S_h at zero mean pipe flow



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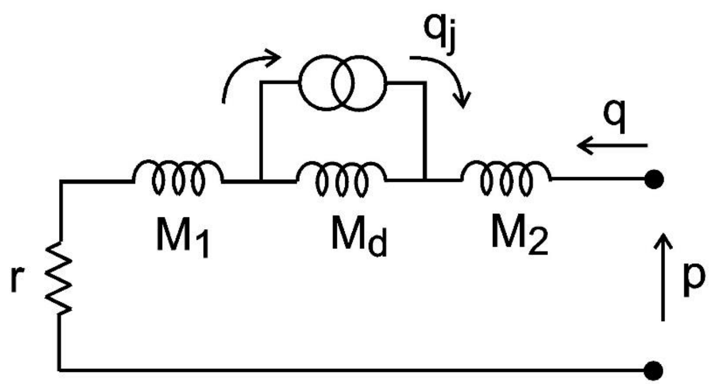
Linear model

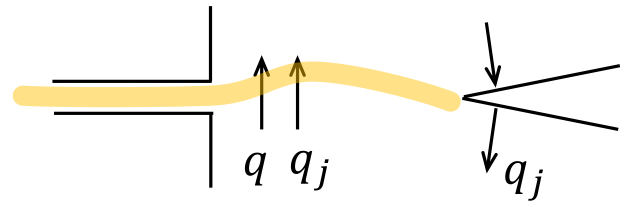
Powell (1953, 1961), Cremer & Ising (1967),

Elder (1973), Fletcher (1976),…

Verge, Hirschberg, Causse, JASA (1997)

Fabre, Hirschberg, Acustica (2000)





$$q_j = J(\omega)(\alpha_p q + \alpha_j q_j)$$

$$Z_h = \frac{p}{q} = r + i\omega M_1 + i\omega M_d \frac{1 + \left(\alpha_p - \alpha_j\right) J(\omega)}{1 - \alpha_j J(\omega)} + i\omega M_2$$

$$S_h = \frac{Z_h - Z_0}{Z_h + Z_0}$$

follow Verge
$$\begin{cases} \alpha_p = 2/\pi \\ \alpha_j = 0.38 \\ M_d = 0.88\rho/H \\ Z_0 = \rho c/(\pi R^2) \end{cases}$$

M is fit at $J = 0$ $\quad M = M_1 + M_d + M_2$

Jet model

Fletcher, JASA (1976)

Nolle, JASA (1998)

jet gain parameter *g*

$$J(\omega) = -g\,\frac{U_o}{W}\,\frac{1}{i\omega}\left(1 - e^{\mu\widetilde{W}} e^{-i\omega\widetilde{W}/c_p} \right)$$

$U(y) = U_0 sech^2(y/b)$ Bickley jet profile with $b = \frac{2}{5}h$ (follow Verge)

$\widetilde{W} = W + d$ Jet path length includes chamfers

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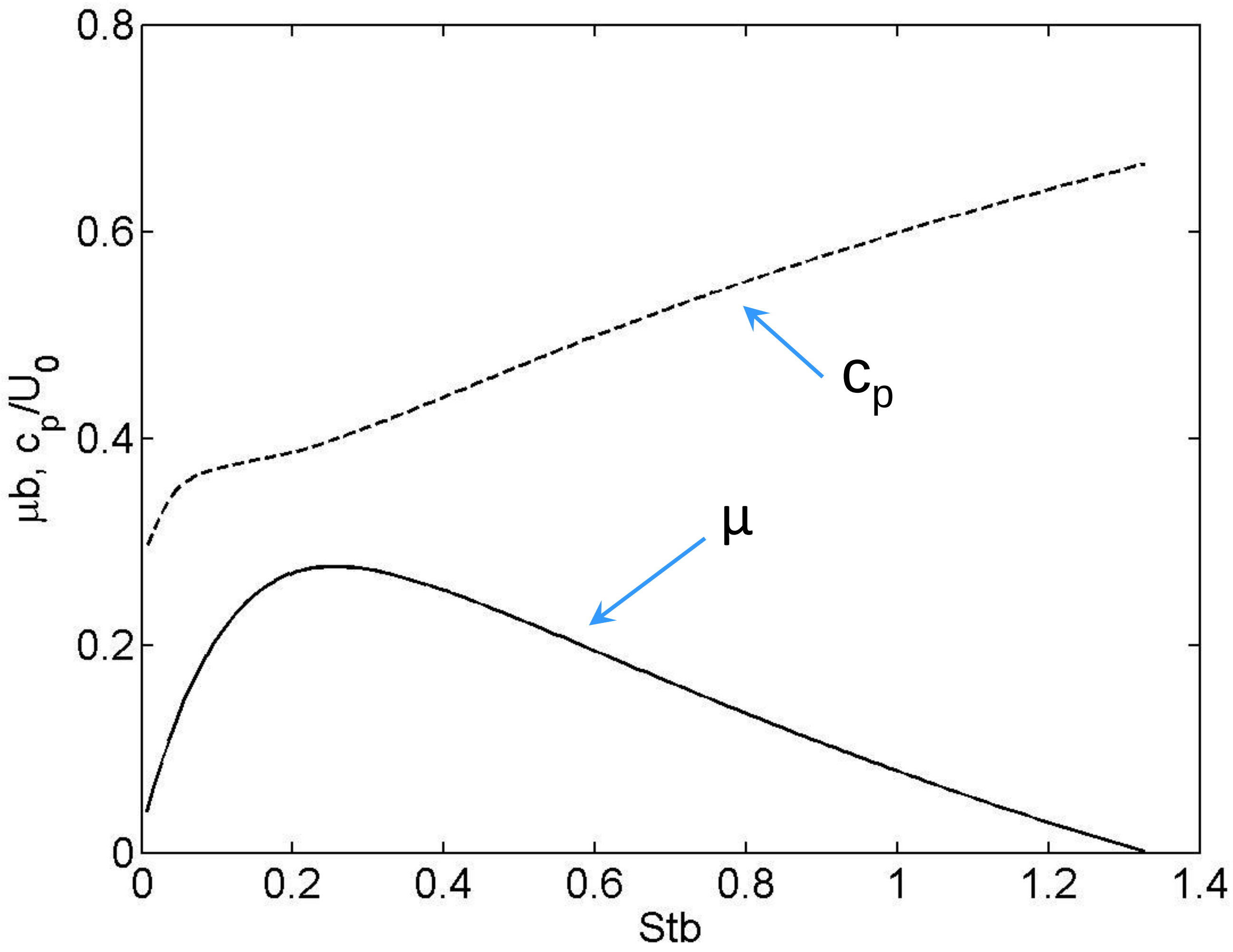
Jet model c_p and μ

Mattingly and Criminale, Phys. Fluids (1971)



$$Stb = \frac{\omega b}{U_0}$$

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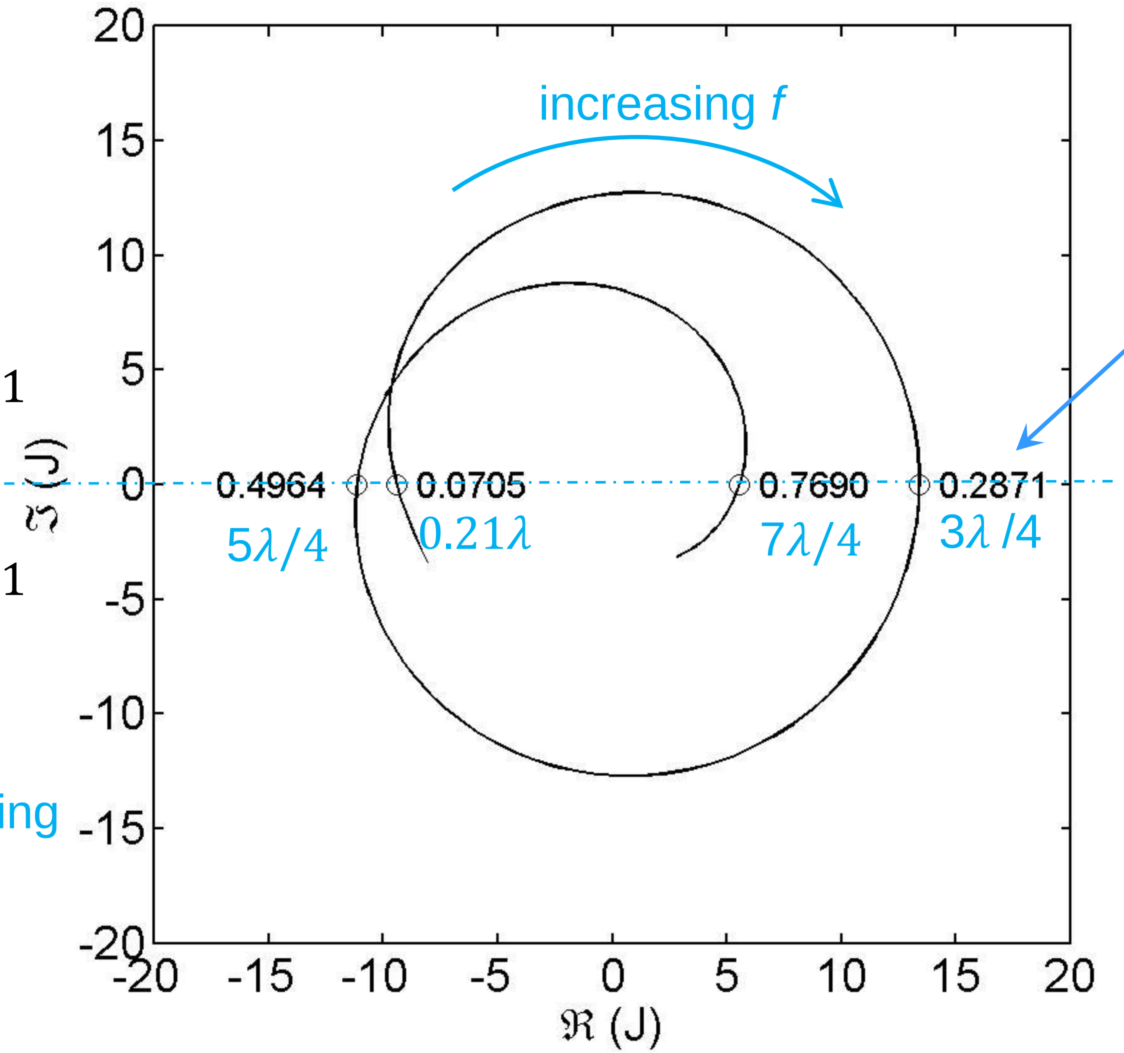
Jet transfer function $J(\omega)$ (for g = 1)

$|S_h|^2 > 1$

$|S_h|^2 < 1$

if not oscillating



$$St = \frac{f\widetilde{W}}{U_0}$$

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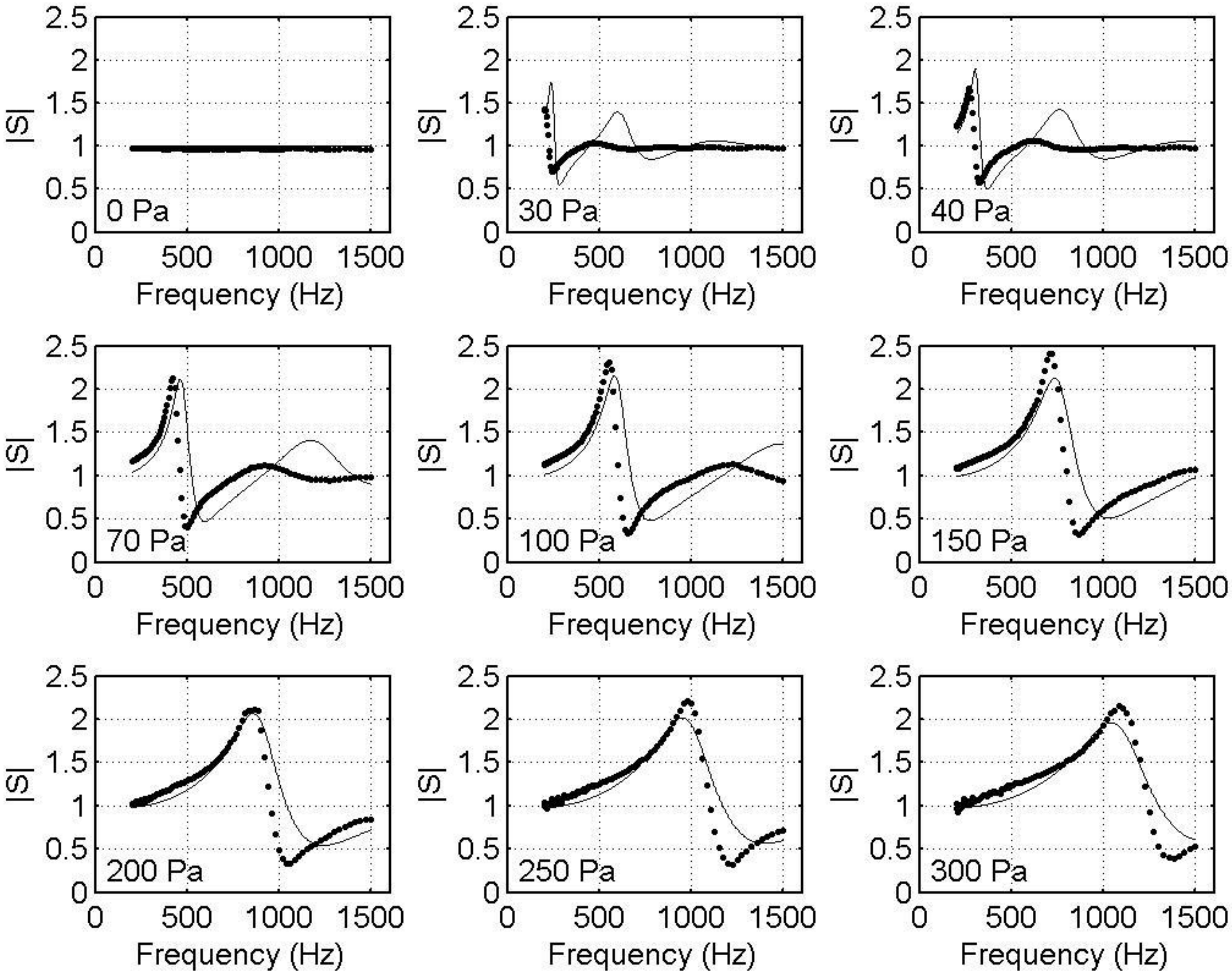
$|S_h|$ at zero mean pipe flow



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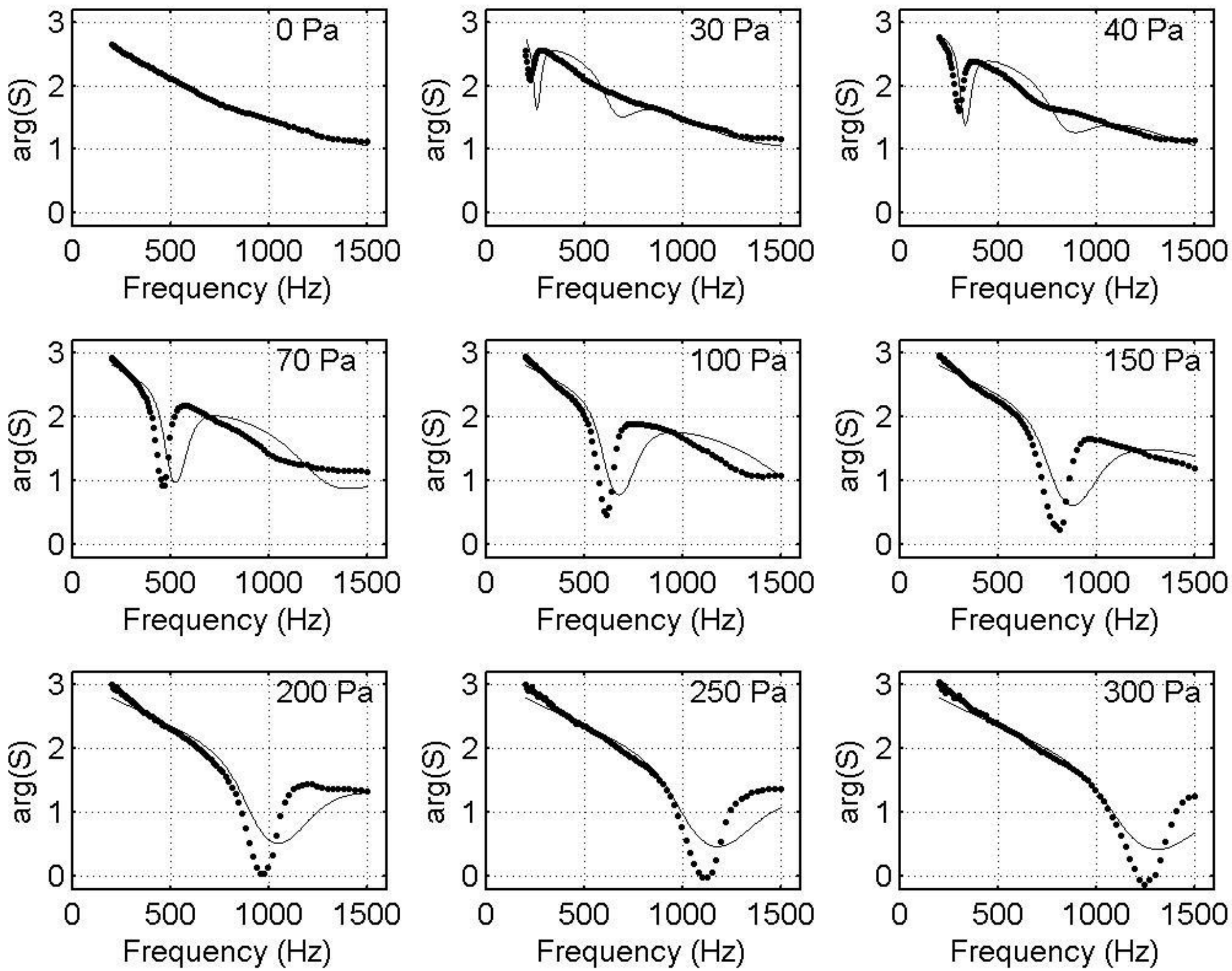
fit parameters: jet gain g = 0.145
total inertance M

Phase of S_h at zero mean pipe flow



fit parameters: jet gain g = 0.145
total inertance M

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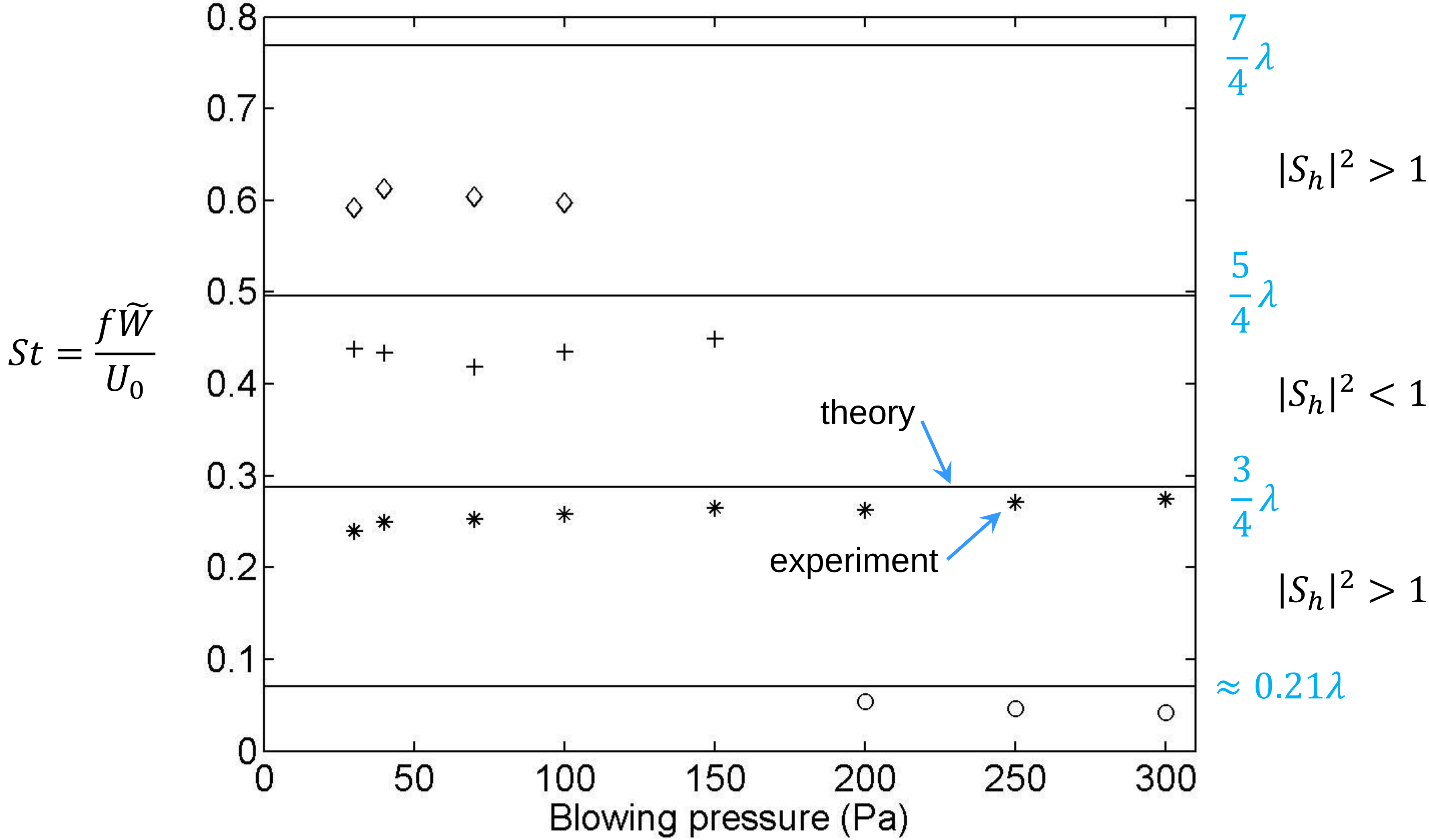
$|S_h|$=1 points at zero mean pipe flow



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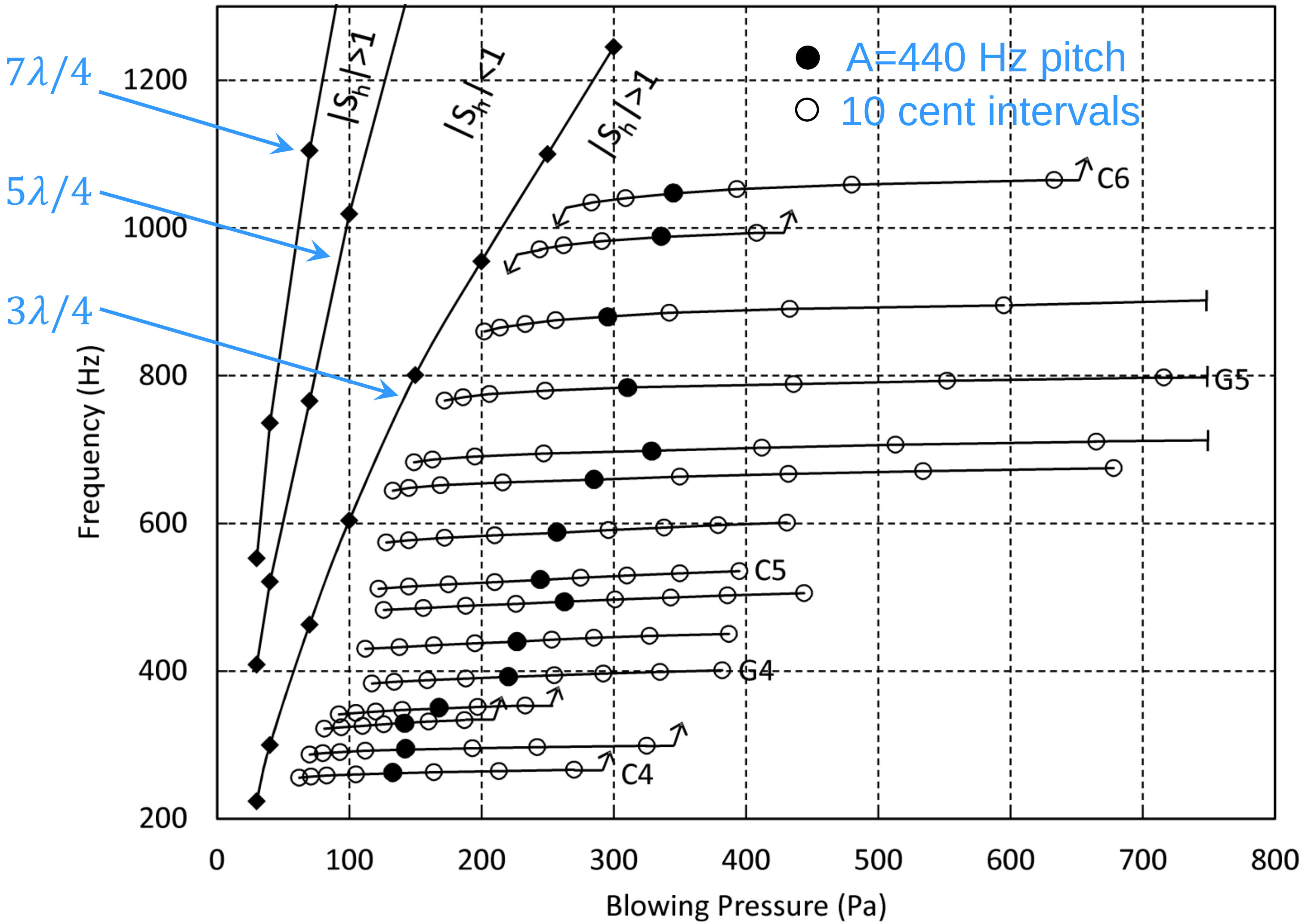
Normal playing region is bounded by measured $|S_h|=1$



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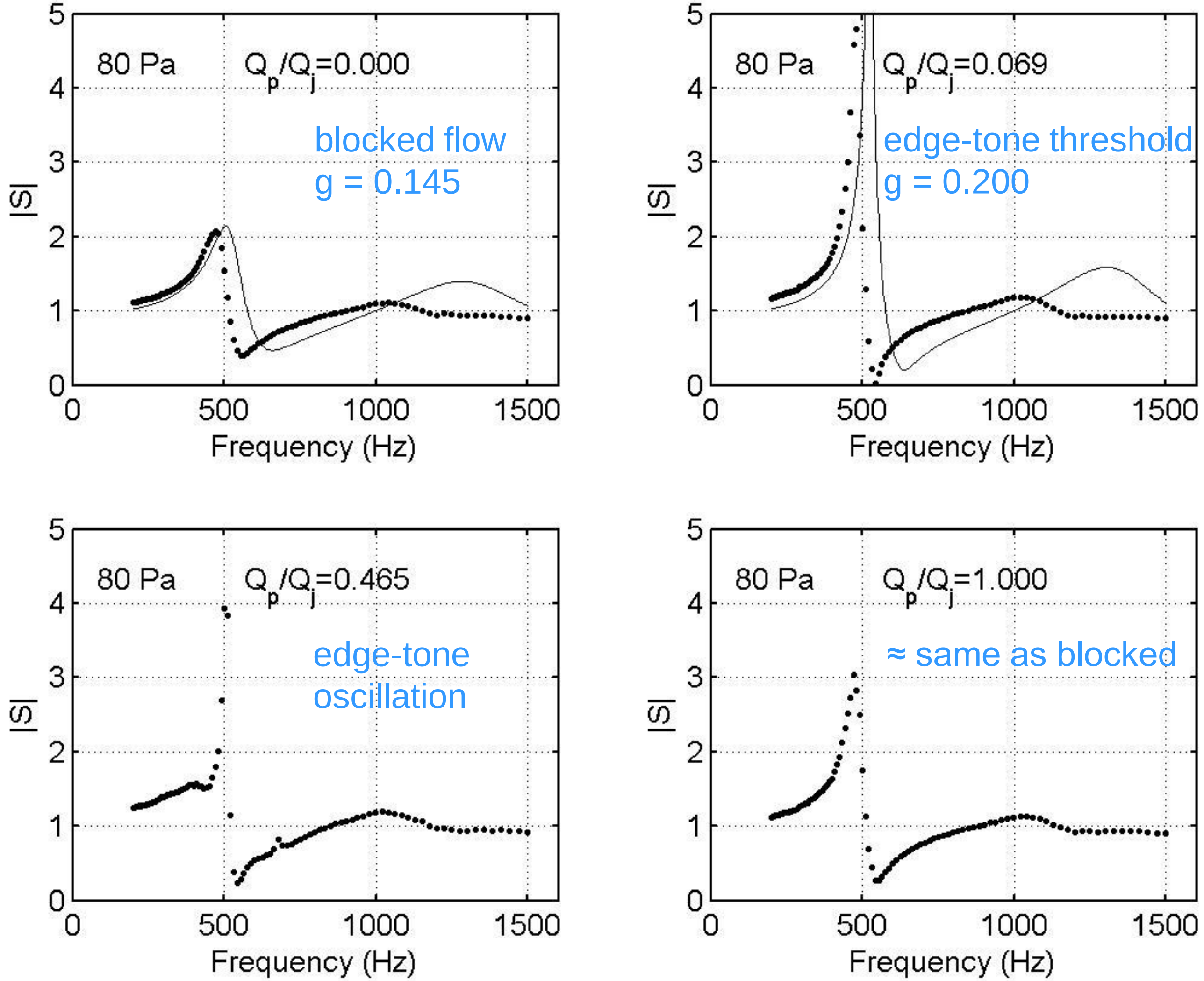
$|S_h|$ versus mean pipe flow



Conclusions

- Linear-response reflection measurements on an unmodified recorder head are possible for mean pipe flow ≈ 0 and for mean pipe flow ≈ jet flow.

- Mean pipe flow can be used to control the jet gain g.

- With an absorbing termination, the head shows edge-tone oscillations for intermediate values of mean flow. The oscillation frequency is very close to the frequency of the gain peak. Edge-tone oscillations disappear at high incident amplitudes.

- The linear model after Verge shows fair agreement with the data. A jet deflection model gives g = 0.58 but data fits g = 0.145. Jet phase velocity c_p is too large at higher St.. Effects of chamfers? Segoufin JASA (2004), Giordano JASA (2014).

- The observed $|S_h| = 1$ boundary at St = 0.27 agrees closely with the low-blowing-pressure boundary for pipe-tone oscillations under normal playing conditions.

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Supplemental Slides

Questions

- Can the linear model be refined for linear-response conditions? (Is the jet gain really too large? Why are the higher $|S_h| = 1$ boundaries at wrong *St*? Can parameter estimates be improved?)

- Can this measurement method be used to characterize gain-saturation? Could we then infer a useful model of the head that could help explain the observed limit cycles?

- Can the linear model be extended to describe pipe-tone and edge-tone saturation behavior?

Some questions in recorder acoustics

Dynamics

Can we model periodic limit cycles vs blowing pressure and fingering?

Can we understand multiphonic limit cycles?

Can we understand why some notes “sound” more easily?

Structure

What is the role of chamfers?

What is the role of windway length and curvature?

What is the role of lip asymmetry?

Why do some instruments “burble”?

Why do fingering patterns of recorders differ from those of traversos?

Modeling

How well do existing lumped models work?

Can gain saturation be understood using lumped models?

What are useful observables for computational experiments?

Kinds of experiments

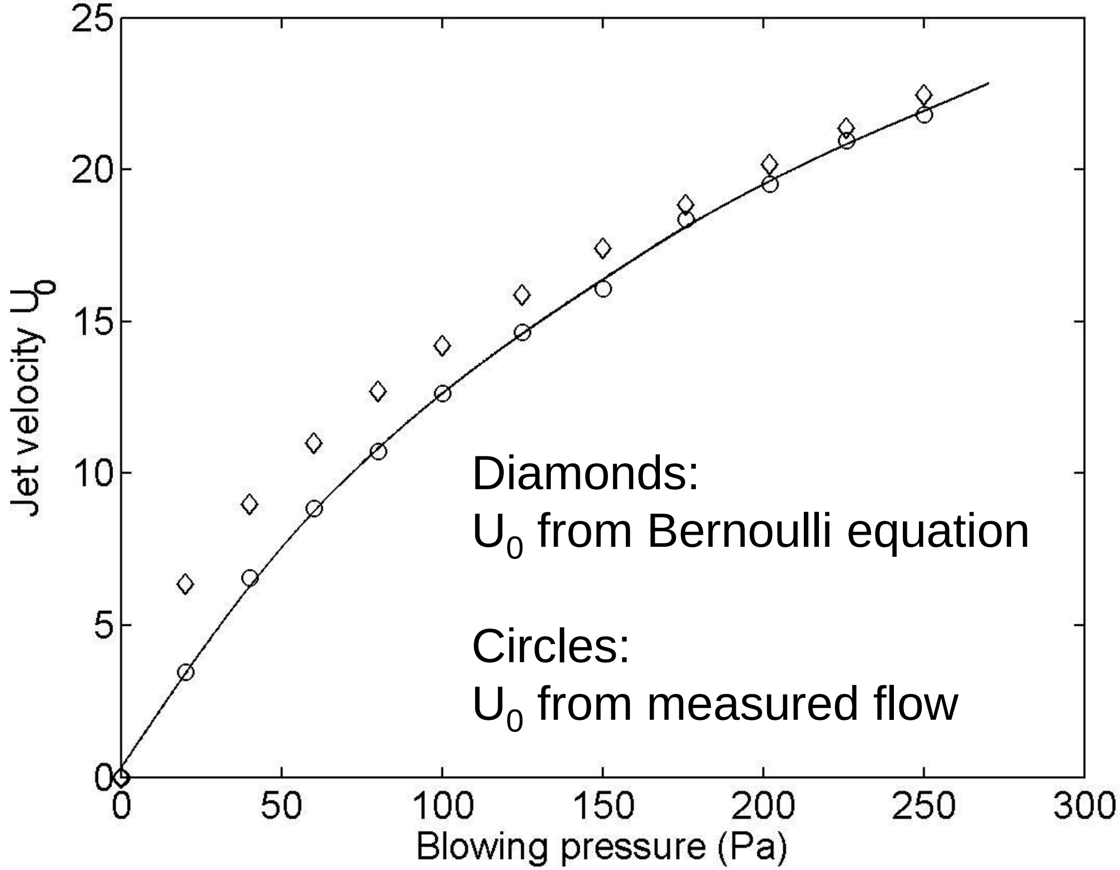
Complete instrument

Radiated steady sound field vs. blowing pressure, geometry	amplifier saturated
Internal steady sound field vs. blowing pressure, geometry	amplifier saturated
Flow visualization	amplifier saturated
Transients	briefly unsaturated

Instrument in parts

Free jets	no amplifier
Embouchure impedance (transverse flute)	no amplifier
Unblown normal modes	no amplifier
Tone hole properties, tone hole arrays	no amplifier

Central jet velocity U_0

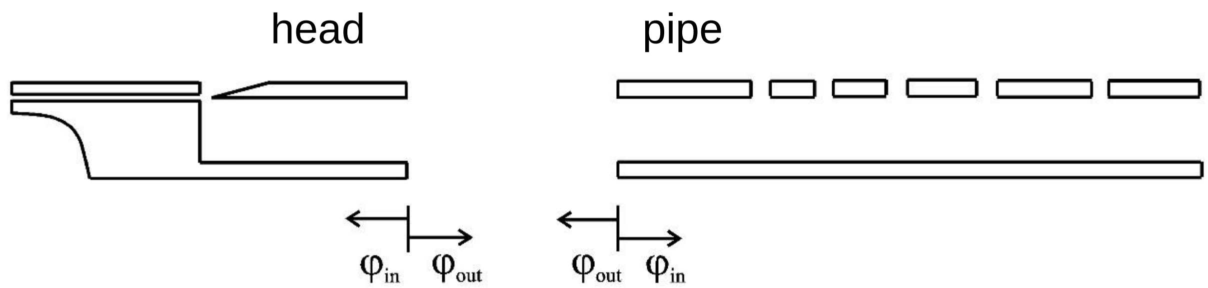


Assume Poiseuille profile at the duct exit.

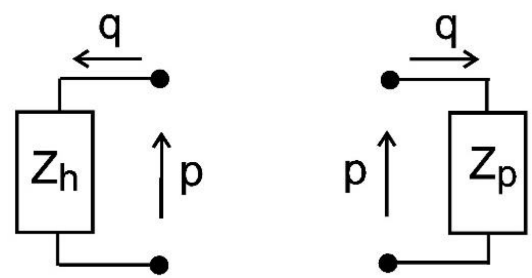
→ must measure jet flow

Reflectometer method I

$S = \varphi_{out}/\varphi_{in}$



$Z = p/q$



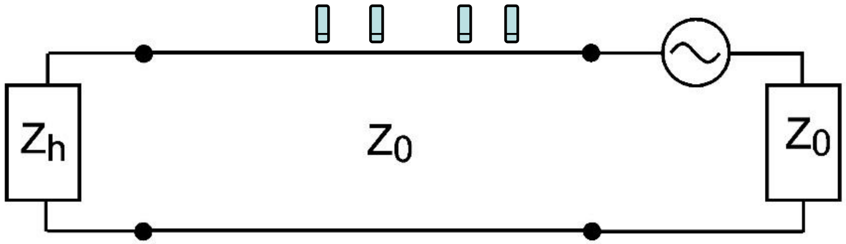
1. When the instrument is assembled $S_h S_p = 1$ or $Z_h + Z_p = 0$.

2. If the system is linear, solutions for real ω give oscillation thresholds.

3. The pipe is passive and almost lossless: $\left|S_p\right|^2 \lesssim 1$. This implies:

Pipe-tone oscillation condition: $|S_h|^2 > 1$.

Reflectometer method II



1. Measure S_h with a matched transmission line and signal source. A microphone array on the line is used to infer φ_{in} and φ_{out}. The experimenter can control the wave amplitude, frequency and the mean pipe flow.

2. With an absorbing termination, the system only oscillates when $S_h \to \infty$

Edge-tone oscillation condition: $S_h \to \infty$.

Coltman JASA (1968): impedance head & tuner, transverse flute, in saturation

Thwaites & Fletcher JASA (1983): "slotted line" SWR method, flue pipe, linear response