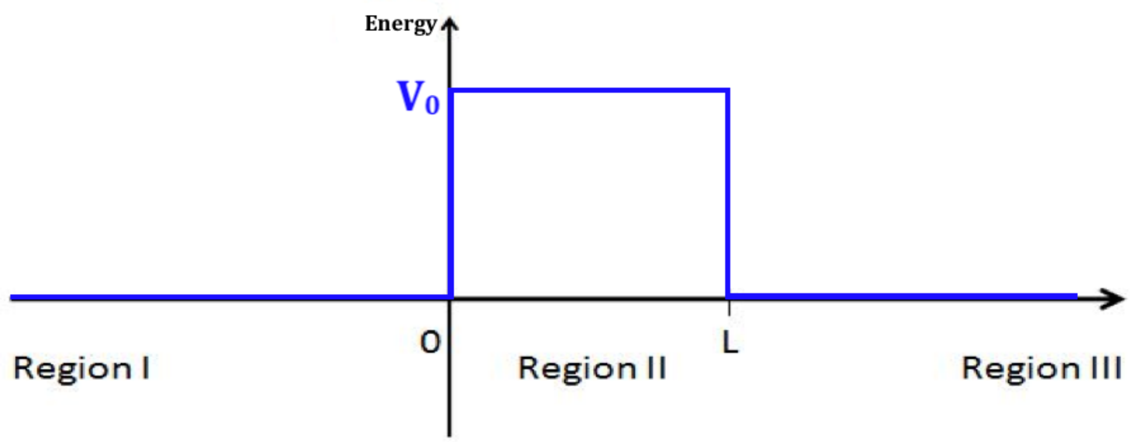
PHYS 3220 PhET Quantum Tunneling Tutorial

Part I: Mathematical Introduction

Recall that the Schrödinger Equation is $i\hbar\frac{\partial\Psi(x,t)}{\partial t} = \hat{H}\Psi(x,t)$. Usually this is solved by first assuming that $\Psi(x,t) = \psi(x)\phi(t)$, from which we obtain the solution $\phi(t) = e^{-iEt/\hbar}$ and are left with the following equation to solve for the spatial dependence:

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}(E - V(x))\psi$$

1. Consider a potential region such as the one shown in the figure below. Given that $E > V_0$, write down a general solution of the Schrödinger Equation for each region. Define any constants that will simplify your solution.

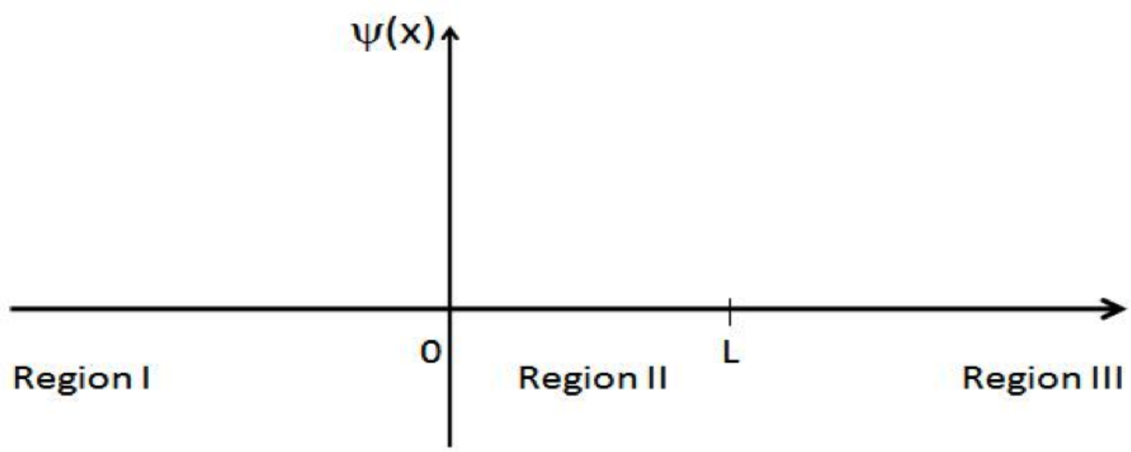


2. How many boundary conditions are needed to completely specify this situation?

3. If a right-going plane wave with amplitude A originating from $x \to -\infty$ is incident upon the barrier, what simplifications can be made in your above equations? Which of your unspecified constants (if any) are now specified completely?

4. What are the remaining boundary conditions for this system? (A simple mathematical formula or explanation in words are both acceptable.)

5. Using this information, do your best to make a plot of the wave function for the case of $E > V_0$.



6. In the graph that you just drew, did you account for the wavelength and amplitude differences in the three regions? (Don't change your graph, just think about it!)
   (a) Rank the magnitude of the wavelengths for the three regions.
   (b) How do you expect the amplitude to compare across the three regions? Give a brief qualitative explanation.

Part II: Plane Wave of $E > V_0$ using PhET sim

Download the Tunneling PhET sim, found at: `http://phet.colorado.edu/sims/quantum-tunneling/quantum-tunneling_en.jar`. Play with the sim for a bit, and then switch to "Plane Wave" mode to answer the following questions.

Notice: For this tutorial, you may find it very useful to switch between using the "Separate" and "Sum" representations on the sim!!!

1. Comparing your findings in Part I to the sim:
   (a) What are the main differences between your plot of the wave function and what is shown?

   (b) Do your predictions for wavelength and amplitude agree with what you see? If not, why were your predictions wrong?

2. You should be able to see the wave function in Region 1 bob up and down.
   (a) What causes this? (You might find the 'Notice' at the top of the page helpful!)

(b) List *all* parameters that you can adjust to eliminate this "bobbiness." Is there only one way to do this, or are there several different ways?

3. Play with the sim and maximize the amount of transmission to Region 3.

   (a) What parameters affect the amount of transmission in this region? List them all. Again, is there only one way to maximize the amount of transmission, or are there multiple ways?

   (b) How does the case of maximum transmission compare to "eliminating the bobbiness" in region 1? Give a brief qualitative explanation of why this is the case.

   (c) Often times the probability of transmission is denoted by the variable T, and takes the following form:

$$T = \frac{1}{1 + \frac{V_0 \sin^2(k_2 L)}{4E(E - V_0)}}$$

   According to this equation, what condition must be satisfied for maximum transmission to occur?

(d) How many variables does the Transmission probability depend on (don't forget to think about what k_2 depends on)? Does this account for everything you found in 3a?

4. Is there any way to set up the sim such that there is a time-dependence in the probability density? Use the fact that $\Psi_{\text{region } j} = A_j e^{i(k_j x - \omega_j t)} + B_j e^{-i(k_j x + \omega_j t)}$ to justify your answer.

5. (a) Based on your result from 4, which regions can show sinusoidal probability densities in the spatial dependence?

(b) Is there any way to make Region 3 have a sinusoidal probability density?

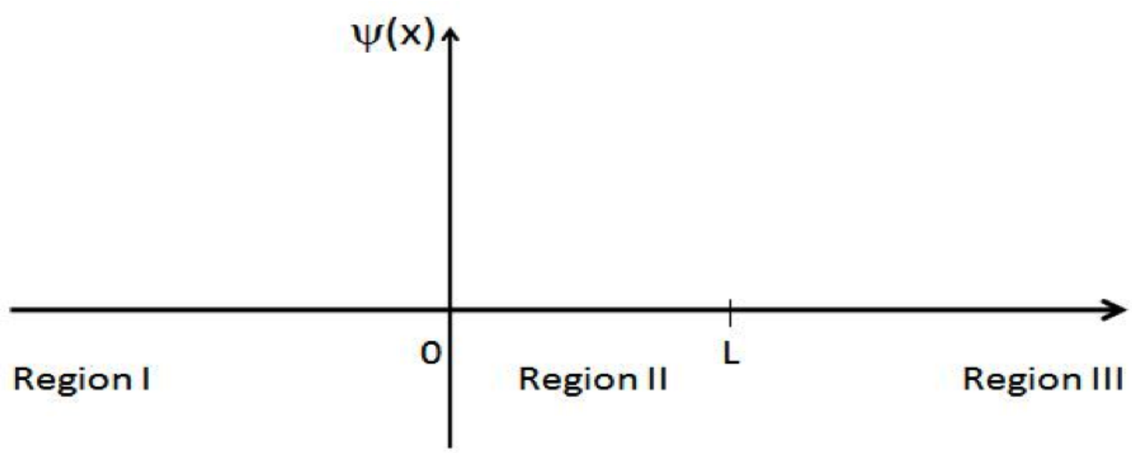
(c) Under what conditions can you have a sinusoidal probability density?

Part III: Case of $E < V_0$

1. For the case of $E < V_0$, write down the most general solutions to the Schrödinger Equation for each of the three regions. Define any constants that help simplify your answers.

2. This time, assume that a left-going plane wave with *fixed amplitude* originating from $x \to +\infty$ is incident upon the barrier. Which variables are now "fixed" or completely specified? List the remaining boundary conditions using a simple mathematical expression.

3. Do your best to plot $\psi(x)$ vs. x across all three regions.



Using the sim for $E < V_0$

1. What parameters can you adjust to maximize the amount of transmission to Region 1 in the sim? List all of the possible ways.

2. (a) When $E < V_0$, is there any way to completely eliminate the reflected wave in Region 3?

   (b) Assuming that the potential barrier, V_0, has *some* finite width, is there any way to get 100% transmission in this case? Why is this the case?

3. Looking at the wave function in the potential barrier, is there any similarity to the case of $E > V_0$, where there was a reflected wave and a transmitted wave? Is the full wave function a sum of solutions or just one particular solution?