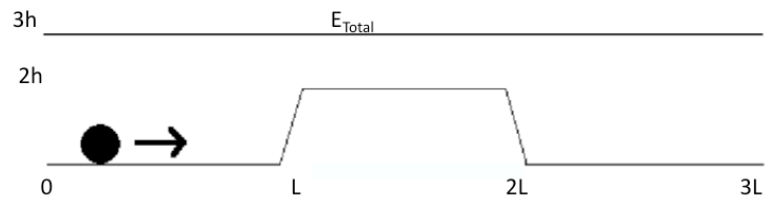
Name:___________________________________ Student ID:___________________________

Tutorial: Quantum Tunneling

PART A: CLASSICAL PARTICLE

A ball of mass *m* rolls to the right on a flat, frictionless surface with total energy $E = 3mgh$. The ball soon encounters a sloped surface and rolls up to height *2h*. After, the ball rolls back down the ramp, always staying in contact with the surface.



1) Is the total energy of the ball as it rolls from 0 to 3L increasing, decreasing, or staying the same?

2) Sketch the kinetic energy, gravitational potential energy, and total energy of the ball between 0 and 3L. Scale your graph with multiples of mgh.

3) Is the amount of time the ball spends between L and 2L greater than, less than, or equal to the amount of time it spends between 0 and L? How does it compare to the amount of time it spends between 2L and 3L? (Ignore the time the ball spends on the ramp.)

4) Now imagine that we take a photograph of the ball at some random time. Is the probability of finding the ball between 0 and L greater than, less than or equal to the probability of finding it between L and 2L? Why?

Name:__________________________________ Student ID:__________________________

PART B: SOLUTIONS TO SCHRÖDINGER'S EQUATION

The time-independent Schrödinger equation is given by:

$$\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + V(x)\psi(x) = E_{TOT}\psi(x)$$

This can be rewritten as:

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}\left(E - V\right)\psi = \frac{2m}{\hbar^2}\left(V - E\right)\psi$$

1) If ***E*** < ***V***, will the solutions to Schrödinger's equation be real exponentials or complex exponentials? [Hint: Is the quantity on the right-hand side positive or negative in this case?]

2) Write down the most general solution to Schrödinger's equation for the case when ***E*** < ***V***.

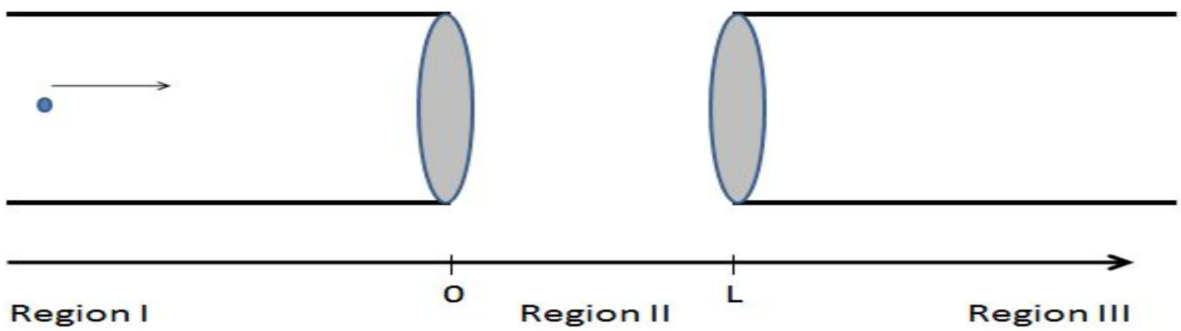
3) If ***E*** > ***V***, will the solutions to Schrödinger's equation be real exponentials or complex exponentials? [Again, consider whether the right-hand side is positive or negative.]

4) Write down the most general solution to Schrödinger's equation for the case when ***E*** > ***V***.

Name:________________________________ Student ID:_________________________

PART C: ELECTRON IN A WIRE ($E > V_0$)

Consider an electron with total energy $\boldsymbol{E}$ moving to the right through a very long smooth copper wire with a ***small*** air gap in the middle:



Assume that the work function of the wire is V_0 and that $V = 0$ inside the wire.

1) If $\mathbf{E > V_0}$, draw a graph of the electron's potential energy in all three regions. Also draw a dashed line indicating the total energy of the electron.

2) In each of the three regions, are the solutions to Schrödinger's equation real exponentials or complex exponentials? Write down a solution for each of the three regions corresponding to an electron traveling to the right.

Region I:

Region II:

Region III:

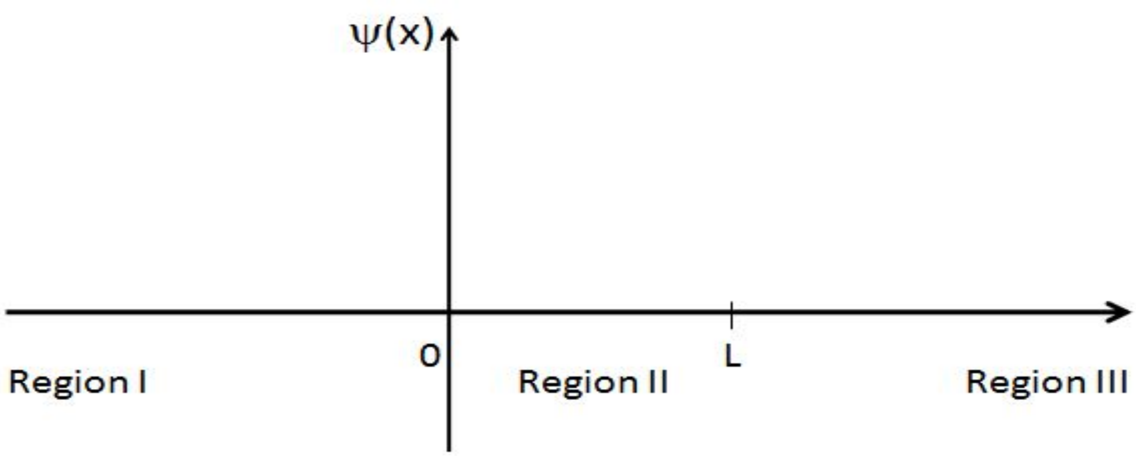
Name:________________________________ Student ID:_________________________

3) How does the kinetic energy of the electron compare in each of the three regions? Rank the kinetic energies in the three regions (***KE_1***, ***KE_2*** & ***KE_3***) from high to low.

4) How does the deBroglie wavelength of the electron compare in each of the three regions? Rank the wavelengths in the three regions (λ_1, λ_2, λ_3) from largest to smallest. If the wavelength is not defined in a particular region, then say so.

5) How does the amplitude of the electron's wave function compare in each of the three regions? [Hint: think about $|\psi(x)|^2$ what tells you in terms of probabilities].

6) With this information in mind, sketch the ***real part*** of the electron's wave function in all three regions:



Name:__________________________________ Student ID:__________________________

PART D: ELECTRON IN A WIRE ($E < V$)

Consider the same situation as in **Part C**, but now the total energy E of the electron is ***less than*** the work function V_0.

1) If $\mathbf{E < V_0}$, draw a graph of the electron's potential energy in all three regions. Also draw a dashed line indicating the total energy of the electron.

2) In each of the three regions, are the solutions to Schrödinger's equation real exponentials or complex exponentials? Write down a solution for each of the three regions corresponding to an electron traveling to the right.

Region I:

Region II:

Region III:

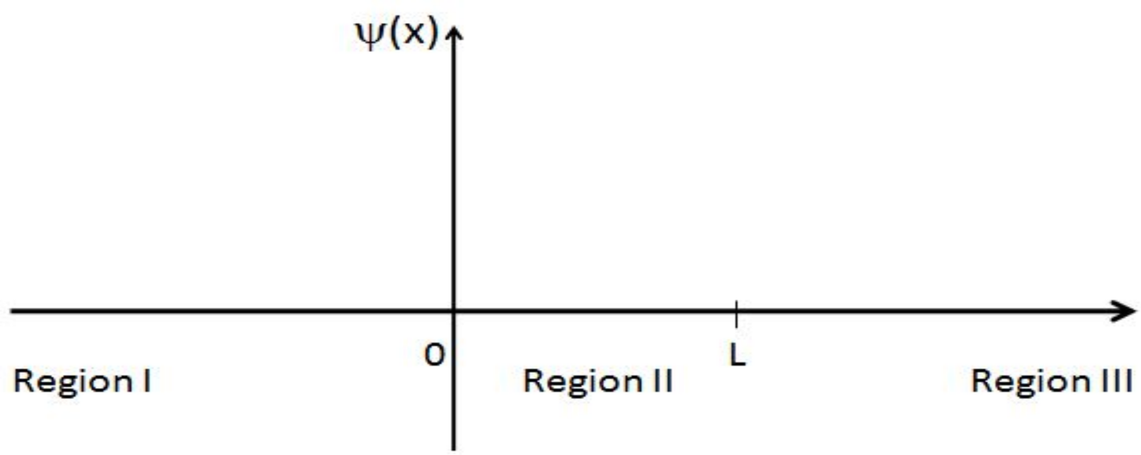
3) How does the kinetic energy of the electron compare in each of the three regions? Rank the kinetic energies in the three regions (KE_1, KE_2 & KE_3) from high to low.

Name:_________________________________ Student ID:__________________________

4) How does the deBroglie wavelength of the electron compare in each of the three regions? Rank the wavelengths in the three regions (λ_1, λ_2, λ_3) from largest to smallest. If the wavelength is not defined in a particular region, then say so.

5) How does the amplitude of the electron's wave function compare in each of the three regions? [Hint: think about $|\psi(x)|^2$ what tells you in terms of probabilities]. Explain what physical meaning we can make from the shape of the wave function in Region II.

6) With this information in mind, sketch the ***real part*** of the wave function for this electron:



Name:_________________________________ Student ID:__________________________

7) Using the solution to **#6**, what conclusions can you make about the possible position of the particle? How is this different than a classical particle in the same situation? Can you offer an explanation of why classical objects (people) don't exhibit the same property, called tunneling?