Abstract Algebra 1 (MATH 3140)

Worksheet 1: Sets, Relations, Functions, and the Natural Numbers

1. Let ρ be a relation on a set A. Show that

(a) ρ is symmetric if and only if $\rho^{-1} \subseteq \rho$, and

(b) ρ is transitive if and only if $\rho \circ \rho \subseteq \rho$.

2. We define a relation ρ on the set of points in $\mathbb{R}^2$ as follows: for any two points $A = (a_1, a_2)$ and $B = (b_1, b_2)$ we let

$$A \,\rho\, B \quad \Longleftrightarrow \quad a_1^2 + a_2^2 = b_1^2 + b_2^2.$$

(a) Show that ρ is an equivalence relation.

(b) Find the partition corresponding to ρ, and describe it geometrically.

3. Prove that for any function $f\colon A \to B$ the following conditions are equivalent:
(i) $f\colon A \to B$ is surjective;
(ii) for arbitrary set C and functions $g, h\colon B \to C$, if $g \circ f = h \circ f$ then $g = h$.

4. Use the definition of $+$ for natural numbers and the Induction Theorem[1] to verify the associative law for $+$:

(‡) $$k + (m + n) = (k + m) + n \qquad \text{for arbitrary natural numbers } k, m, n.$$

Hint: Use induction on n. Start by defining the set S to which you want to apply the Induction Theorem.

[1]See the handout "The Natural Numbers and Induction".