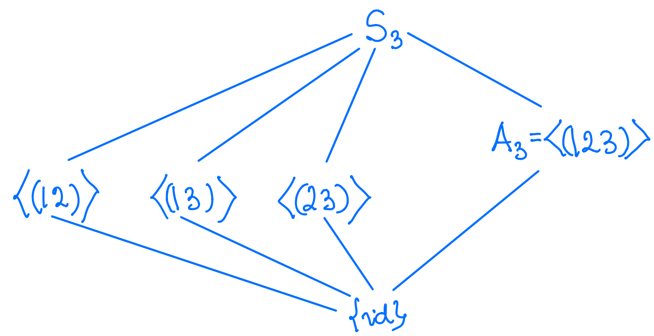
Abstract Algebra 1 (MATH 3140)

Worksheet 3: Normal Subgroups and Conjugation. Lagrange's Theorem

1. Use Lagrange's Theorem to verify that S_3 has no other subgroups than those in the inclusion diagram here.



2. (a) Find all conjugacy classes of S_3.

(b) Use the result in part (a) to find all normal subgroups of S_3.

3. Let H be a subgroup of a group G. Show that if $[G : H] = 2$, then H is a normal subgroup of G.

Hint: Argue that $aH = Ha$ for all $a \in G$. Distinguish two cases: $a \in H$ and $a \notin H$.

4. Recall that the alternating group A_4 is the subgroup of S_4 consisting of all even permutations in S_4.

(a) Show that A_4 is a normal subgroup of S_4, and $|A_4| = 12$.

(b) List the elements of A_4, and find all conjugacy classes of A_4.

(c) Use the results of part (b) and of Problem 3 to prove that A_4 has no subgroup of order 6.[1]

(d) Find all subgroups of A_4, and draw their inclusion diagram. Which subgroups of A_4 are normal?

[1]**Remark:** Let G be a finite group. By Lagrange's theorem, if G has a subgroup of order d, then $d \,\big|\, |G|$. The result of this exercise shows that the converse of this statement is not true: for $G = A_4$ and $d = 6$ we have $d \,\big|\, |G|$, but G does not have a subgroup of order d.