Abstract Algebra 1 (MATH 3140)

Worksheet 4: Quotient Groups and the Homomorphism/Isomorphism Theorems

1. Let $G = \langle a \rangle$ be a cyclic group of order n, and let $d \mid n$ ($d \in \mathbb{N}$). Show that the subgroup $N = \langle a^d \rangle$ of G is normal, and G/N is a cyclic group of order d.

2. Find two subgroups M, N of A_4 such that $M \trianglelefteq N \trianglelefteq A_4$, but $M \ntrianglelefteq A_4$.

3. Let G be a finite group, and let H be a subgroup of G of index 2. Prove that every element of G that is not in H has even order.
Hint: Use a quotient group of G.

4. Let G be a finite group, and let $H \leq G$, $N \trianglelefteq G$. Show that the subgroup HN of G has order $\dfrac{|H||N|}{|H \cap N|}$.

Hint: Use the Diamond Isomorphism Theorem.

5. Does D_6 have a normal subgroup N such that $D_6/N \cong D_3$?