PHILOSOPHY 4360/5360 - METAPHYSICS

Handout: Achilles and the Tortoise Calculation

Stage of the Race	Achilles' Position	The Tortoise's Position	Time to Travel from the Previous Position to the Present Position in Hours
0	0	10 miles	0
1	10 miles	15 miles	1/2 an hour
2.	15 miles	17.5 miles	1/4 an hour
3	17.5 miles	18.75 miles	1/8 an hour
4.	18.75 miles	19.375 miles	1/16 an hour
5.	19.375 miles	19.6875	1/32 an hour
6.	19.6875	19.84375	1/64 an hour
7.	19.84375	19.921875	1/128 an hour

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To work how long this process with an infinite number of steps takes, then, one needs to add up the infinite (geometrical) series that starts off

1/2 + 1/4 + 1/8 + 1/16 +1/32 + 1/64 + 1/128 . . .

3. How can this be done? Here is a simple way of summing the infinite geometrical series in question. (By generalizing this method one can derive a formula that one can use for any infinite geometrical series.)

Let T = 1/2 +1/4 + 1/8 + 1/16 + 1/32 + 1/64 + 1/128 + etc.

Now multiple both the left hand side and every term in the right hand side of this equation by two, so that one has:

2T = 1 + 1/2 +1/4 + 1/8 + 1/16 + 1/32 + 1/64 + 1/128 + etc.

Now place the first equation under the second, shifting the terms on the right hand side of the first equation a bit to the right, so they fall under equal terms in the top equation, as follows:

2T = 1 + 1/2 +1/4 + 1/8 + 1/16 + 1/32 + 1/64 + 1/128 + etc.

T = 1/2 +1/4 + 1/8 + 1/16 + 1/32 + 1/64 + 1/128 + etc.

One can now see that when one subtracts the first equation from the second, one has

T = 1

So the infinite process in the example of Achilles and the tortoise described above will take one hour, and during that time Achilles will cover 20 miles while the tortoise will cover 10 miles. Since the tortoise had a 10-mile head start, Achilles has caught up to the tortoise after exactly one hour.