Prob. 1

First, let's find the failure criterion for the material:

a) In the principal space:

b) In the $\sigma - \tau$ space:

The torsion creates only shear stress and the axial load only creates normal stress.

\[
\tau = \frac{Tr}{I_p} \quad I_p = \frac{\pi r^4}{2} = \frac{\pi \times 0.75^4}{2} = 0.497\text{in}^4, \quad \tau = \frac{13.3\times 0.75}{0.497} = 20.07\text{ksi}
\]

\[
\sigma = \frac{P}{A} = \frac{P}{\pi r^2} = 0.566P, \quad [\sigma] = \text{ksi}, \quad [P] = \text{kip}
\]

- **Method 1:**

  Let's look at the Mohr circle, i.e. in the $\sigma - \tau$. We have $\sigma_x = 0.566P$, $\sigma_y = 0$, $\tau = 20.07\text{ksi}$:
The safety factor is 1.22: means that if the current stress state is scaled by 1.22, the new Mohr circle would touch the envelope and yield:

\[
\frac{R}{c + 33.9} = \sin(35.7^\circ)
\]

Radius of the Mohr circle:

\[
R = \sqrt{24.49^2 + \left(\frac{1.22\sigma}{2}\right)^2} = \sqrt{599.76 + 0.372\sigma^2}
\]

Center of the Mohr circle:

\[
c = -\frac{1.22\sigma}{2} = -0.61\sigma \quad \text{(negative sign because } \sigma \text{ is a negative number while } c \text{ is a distance)}
\]

So,
The result is an imaginary number! This means that the safety factor cannot possibly be 1.22! Let's take a look:

\[
\sqrt{599.76 + 0.372\sigma^2} \div (-0.61\sigma + 33.9) = \sin(35.7^\circ) = 0.583
\]
\[
\Rightarrow \sqrt{599.76 + 0.372\sigma^2} = -0.356\sigma + 19.76
\]
\[
\Rightarrow 599.76 + 0.372\sigma^2 = 0.127\sigma^2 - 14.07\sigma + 390.46
\]
\[
\Rightarrow 0.245\sigma^2 + 14.07\sigma + 209.3 = 0
\]

This means that even if \( P = 0 \), the safety factor is about \( \frac{24.35}{20.07} = 1.21 \).

- **Method 2:**
  Solving in the principal stress space:
  \[
  \sigma_1 = 0.5\sigma + \sqrt{\tau^2 + 0.25\sigma^2}
  \]
  \[
  \sigma_2 = 0.5\sigma - \sqrt{\tau^2 + 0.25\sigma^2}
  \]
  So \( \sigma_1 \) is positive and \( \sigma_2 \) is negative, so we are looking at the forth quadrant:

To find \( \sigma_1^{\text{Yield}} \) we must intersect the line \( \sigma_2 = \left[ \frac{0.5\sigma - \sqrt{\tau^2 + 0.25\sigma^2}}{0.5\sigma + \sqrt{\tau^2 + 0.25\sigma^2}} \right] \sigma_1 \) with the yield envelope of the forth quadrant, \( \sigma_2 = \frac{95}{25} \sigma_1 - 95 \):
\[
\begin{bmatrix}
0.5\sigma - \sqrt{\tau^2 + 0.25\sigma^2} \\
0.5\sigma + \sqrt{\tau^2 + 0.25\sigma^2}
\end{bmatrix}
\sigma_{\text{Yield}}^1 = 3.8\sigma_{\text{Yield}}^1 - 95
\]

\[\Rightarrow \sigma_{\text{Yield}}^1 = \frac{95}{3.8 - \left(\frac{0.5\sigma - \sqrt{\tau^2 + 0.25\sigma^2}}{0.5\sigma + \sqrt{\tau^2 + 0.25\sigma^2}}\right)}\]

The safety factor tells us that \(\frac{\sigma_{\text{Yield}}^1}{\sigma_1} = 1.22\), so:

\[\frac{95}{3.8 - \left(\frac{0.5\sigma - \sqrt{\tau^2 + 0.25\sigma^2}}{0.5\sigma + \sqrt{\tau^2 + 0.25\sigma^2}}\right)} = 1.22\]

\[\Rightarrow \frac{95}{3.8 \left(0.5\sigma + \sqrt{\tau^2 + 0.25\sigma^2}\right) - \left(0.5\sigma - \sqrt{\tau^2 + 0.25\sigma^2}\right)} = 1.22\]

\[\Rightarrow \frac{95}{1.4\sigma + 4.8\sqrt{\tau^2 + 0.25\sigma^2}} = 1.22\]

\[\Rightarrow 1.4\sigma + 4.8\sqrt{402.8 + 0.25\sigma^2} = 77.87\]

\[\Rightarrow 402.8 + 0.25\sigma^2 = [16.22 - 0.292\sigma]^2 = 263.09 - 9.47\sigma + 0.085\sigma^2\]

\[\Rightarrow 0.165\sigma^2 + 9.47\sigma + 139.71 = 0\]

Again, the solution is an imaginary number!

**Summary:**
A prudent solution would find that the problem description is not possible: only with the applied torsion, the factor of safety is already below 1.22. Adding an extra P would only make it worse and reduce the factor of safety.
Prob. 2

(a) In general, 3D Mohr’s circles

When $\sigma_1 = 0$

$\sigma_2 = \sigma_3$

From test data, the failure envelop

$\sigma_2 = \sigma_3 = 25000$ psi, same as 2D compression

(b) $\sigma_1 = \sigma_2 = -15000$, $\sigma_3 = ?$

The Mohr’s circles are

Combine the Mohr’s circle with the failure envelop
\[ \frac{c}{a} = \tan \theta \]
\[ a = \frac{c}{\tan \theta} \]

Center = \(\frac{c_1 + c_2}{2}\)

\[ \gamma = \frac{c_1 - c_2}{2} \]

\[ \gamma = \frac{c_1 - c_2}{2} - \gamma_{\text{center}} - A \]

Both \(a\) and \(c_{\text{center}}\)

Add \(-\gamma\) to both sides

\(c_{\text{center}} = \frac{c_1 - c_2}{2} + \frac{c}{\tan \theta} \]

Solve for \(c_2\):

\[ \frac{c_1 - c_2}{2} = \gamma_{\text{center}} + A \]

\[ c_2 = \frac{c_1 + c_2}{2} - \gamma_{\text{center}} - A \]

\[ c = t \tan(90 - \theta) = \frac{\tan \theta}{\cos \theta} \]

\[ c = \frac{t \tan(90 - \theta)}{\cos \theta} = \frac{\tan \theta}{\cos \theta} \]

\[ t \tan \theta = 0.106 \]

\[ \theta_{\text{center}} = 1.27 \]

\[ C = \frac{t^2 \tan(90 - \theta)}{2} \]

\[ C = \frac{t^2 \tan(90 - \theta)}{2} = \frac{t^2 \tan(90 - 0.106)}{2} \]

\[ = 58.4 \text{ psi} \]

\[ \text{at composition} \]
Prob. 3(a)

\[ \sigma = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \]

\[ \sigma_{\text{act}} = \frac{1}{3} \sigma \]

\[ T_{\text{act}} = \frac{\sqrt{2}}{3} \sigma \]

\[ \cos \phi_0 = \frac{\sigma}{\frac{\sqrt{2}}{3} \sigma} = \frac{1}{2} \]

\[ \phi_0 = 60^\circ \]

\[ \frac{\sqrt{2}}{3} \sigma \frac{s_{300}}{s_{300}} - 0.686 \left[ 0.226 + 0.736 \left( \frac{\sqrt{2}}{3} \sigma \frac{s_{300}}{s_{300}} \right) \right] = 0 \]

\[ 2.6 \times 10^{-5} \sigma = 0.153 \]

\[ \therefore \sigma = 5753 \text{ psi} \]

\[ \frac{\sigma_{\text{act}}}{\sigma_0} = \frac{\frac{1}{3} \times 5753}{s_{300}} < 1.25 \checkmark \]

\[ \sigma_1 = \sigma_3 = 5753 \text{ in compression} \]

Prob. 3(b)

Consider compression as positive.

\[ \sigma_{\text{act}} = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \]

\[ T_{\text{act}} = \frac{1}{3} \left[ (\sigma_1 - 3000)^2 + (1500 - \sigma_3)^2 \right]^\frac{1}{2} = \frac{\sqrt{2}}{3} (\sigma_1 - 1500) \]

\[ \cos \phi_0 = \frac{2\sigma_1 - 3000}{3\sqrt{2} (\frac{1}{3} (\sigma_1 - 1500))} = 1 \]

\[ \phi_0 = 0 \]

\[ \frac{\sqrt{2}}{3} \left( \frac{\sigma_1 - 1500}{s_{300}} \right) - 0.686 \left[ 0.226 + 0.736 \left( \frac{\sqrt{2}}{3} \cdot \frac{s_{300}}{s_{300}} \right) \right] = 0 \]

\[ 9.42 \times 10^{-5} \sigma_1 - 9.41 = 0 \]

\[ 4.52 \times 10^{-5} \sigma_1 = 0.5147 \]

\[ \sigma_1 = 11384 \text{ psi} \]

\[ \sigma_{\text{act}} = \frac{1}{3} (11384 + 3000) = 4758 \]

\[ \frac{\sigma_{\text{act}}}{\sigma_0} = 0.859 < 1.25 \checkmark \]

\[ \sigma_1 = 11384 \text{ psi} \]

\[ \sigma = 1500 \text{ in compression} \]